

The formulas of a standard helical rack are similar to those of **Table 6-6** with only the normal coefficient of profile shift  $x_n=0$ . To mesh a helical gear to a helical rack, they must have the same helix angle but with opposite hands.

The displacement of the helical rack,  $l$ , for one rotation of the mating gear is the product of the radial pitch,  $p_t$ , and number of teeth.

$$l = \frac{\pi m_n}{\cos \beta} z = p_t z \quad (6-13)$$

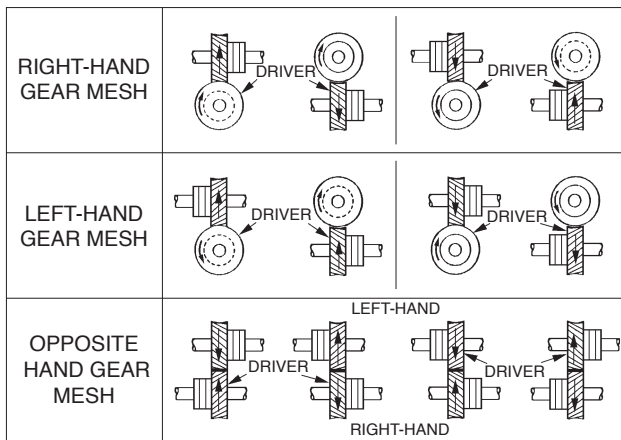
According to the equations of **Table 6-7**, let radial pitch  $p_t = 8$  mm and displacement  $l = 160$  mm. The radial pitch and the displacement could be modified into integers, if the helix angle were chosen properly.

In the axial system, the linear displacement of the helical rack,  $l$ , for one turn of the helical gear equals the integral multiple of radial pitch.

$$l = \pi z m_t \quad (6-14)$$

## SECTION 7 SCREW GEAR OR CROSSED HELICAL GEAR MESHES

These helical gears are also known as spiral gears. They are true helical gears and only differ in their application for interconnecting skew shafts, such as in **Figure 7-1**. Screw gears can be designed to connect shafts at any angle, but in most applications the shafts are at right angles.



**Fig. 7-1 Types of Helical Gear Meshes**

### NOTES:

1. Helical gears of the same hand operate at right angles.
2. Helical gears of opposite hand operate on parallel shafts.
3. Bearing location indicates the direction of thrust.

## 7.1 Features

### 7.1.1 Helix Angle And Hands

The helix angles need not be the same. However, their sum must equal the shaft angle:

$$\beta_1 + \beta_2 = \Sigma \quad (7-1)$$

where  $\beta_1$  and  $\beta_2$  are the respective helix angles of the two gears, and  $\Sigma$  is the shaft angle (the acute angle between the two shafts when viewed in a direction paralleling a common perpendicular between the shafts).

Except for very small shaft angles, the helix hands are the same.

## 7.1.2 Module

Because of the possibility of different helix angles for the gear pair, the radial modules may not be the same. However, the normal modules must always be identical.

## 7.1.3 Center Distance

The pitch diameter of a crossed-helical gear is given by **Equation (6-7)**, and the center distance becomes:

$$a = \frac{m_n}{2} \left( \frac{Z_1}{\cos \beta_1} + \frac{Z_2}{\cos \beta_2} \right) \quad (7-2)$$

Again, it is possible to adjust the center distance by manipulating the helix angle. However, helix angles of both gears must be altered consistently in accordance with **Equation (7-1)**.

## 7.1.4 Velocity Ratio

Unlike spur and parallel shaft helical meshes, the velocity ratio (gear ratio) cannot be determined from the ratio of pitch diameters, since these can be altered by juggling of helix angles. The speed ratio can be determined only from the number of teeth, as follows:

$$\text{velocity ratio} = i = \frac{Z_1}{Z_2} \quad (7-3)$$

or, if pitch diameters are introduced, the relationship is:

$$i = \frac{Z_1 \cos \beta_2}{Z_2 \cos \beta_1} \quad (7-4)$$

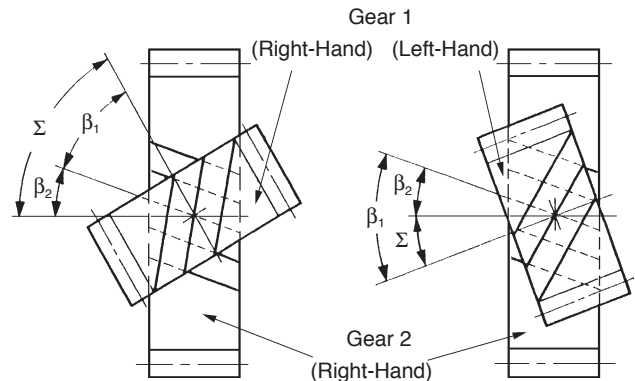
## 7.2 Screw Gear Calculations

Two screw gears can only mesh together under the conditions that normal modules,  $m_{n1}$ , and,  $m_{n2}$ , and normal pressure angles,  $\alpha_{n1}$ ,  $\alpha_{n2}$ , are the same. Let a pair of screw gears have the shaft angle  $\Sigma$  and helical angles  $\beta_1$  and  $\beta_2$ :

$$\left. \begin{array}{l} \text{If they have the same hands, then:} \\ \Sigma = \beta_1 + \beta_2 \\ \text{If they have the opposite hands, then:} \\ \Sigma = \beta_1 - \beta_2, \text{ or } \Sigma = \beta_2 - \beta_1 \end{array} \right\} \quad (7-5)$$

If the screw gears were profile shifted, the meshing would become a little more complex. Let  $\beta_{w1}$ ,  $\beta_{w2}$  represent the working pitch cylinder;

$$\left. \begin{array}{l} \text{If they have the same hands, then:} \\ \Sigma = \beta_{w1} + \beta_{w2} \\ \text{If they have the opposite hands, then:} \\ \Sigma = \beta_{w1} - \beta_{w2}, \text{ or } \Sigma = \beta_{w2} - \beta_{w1} \end{array} \right\} \quad (7-6)$$



**Fig. 7-2 Screw Gears of Nonparallel and Nonintersecting Axes**

**Table 7-1** presents equations for a profile shifted screw gear pair. When the normal coefficients of profile shift  $x_{n1} = x_{n2} = 0$ , the equations and calculations are the same as for standard gears.

Standard screw gears have relations as follows:

$$\left. \begin{aligned} d_{w1} &= d_1, d_{w2} = d_2 \\ \beta_{w1} &= \beta_1, \beta_{w2} = \beta_2 \end{aligned} \right\} \quad (7-7)$$

### 7.3 Axial Thrust Of Helical Gears

In both parallel-shaft and crossed-shaft applications, helical gears develop an axial thrust load. This is a useless force that loads gear teeth and bearings and must accordingly be considered in the housing and bearing design. In some special instrument designs, this thrust load can be utilized to actuate face clutches, provide a friction drag, or other special purpose. The magnitude of the thrust load depends on the helix angle and

is given by the expression:

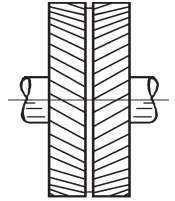
$$W_T = W^t \tan \beta \quad (7-8)$$

where

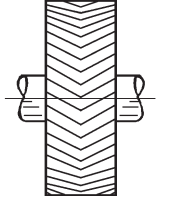
$W_T$  = axial thrust load, and  
 $W^t$  = transmitted load.

The direction of the thrust load is related to the hand of the gear and the direction of rotation. This is depicted in **Figure 7-1**. When the helix angle is larger than about 20°, the use of double helical gears with opposite hands (**Figure 7-3a**) or herringbone gears (**Figure 7-3b**) is worth considering.

More detail on thrust force of helical gears is presented in **SECTION 16**.



**Figure 7-3a**



**Figure 7-3b**

**Table 7-1 The Equations for a Screw Gear Pair on Nonparallel and Nonintersecting Axes in the Normal System**

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Normal Module	$m_n$		3	
2	Normal Pressure Angle	$\alpha_n$		20°	
3	Helix Angle	$\beta$		20°	30°
4	Number of Teeth & Helical Hand	$z_1, z_2$		15 (R)	24 (L)
5	Number of Teeth of an Equivalent Spur Gear	$z_v$	$\frac{z}{\cos^3 \beta}$	18.0773	36.9504
6	Radial Pressure Angle	$\alpha_t$	$\tan^{-1} \left( \frac{\tan \alpha_n}{\cos \beta} \right)$	21.1728°	22.7959°
7	Normal Coefficient of Profile Shift	$x_n$		0.4	0.2
8	Involute Function $\alpha_{wn}$	$\text{inv} \alpha_{wn}$	$2 \tan \alpha_n \left( \frac{x_{n1} + x_{n2}}{z_{v1} + z_{v2}} \right) + \text{inv} \alpha_n$	0.0228415	
9	Normal Working Pressure Angle	$\alpha_{wn}$	Find from Involute Function Table	22.9338°	
10	Radial Working Pressure Angle	$\alpha_{wt}$	$\tan^{-1} \left( \frac{\tan \alpha_{wn}}{\cos \beta} \right)$	24.2404°	26.0386°
11	Center Distance Increment Factor	$y$	$\frac{1}{2} (z_{v1} + z_{v2}) \left( \frac{\cos \alpha_n}{\cos \alpha_{wn}} - 1 \right)$	0.55977	
12	Center Distance	$a_x$	$\left( \frac{z_1}{2 \cos \beta_1} + \frac{z_2}{2 \cos \beta_2} + y \right) m_n$	67.1925	
13	Pitch Diameter	$d$	$\frac{z m_n}{\cos \beta}$	47.8880	83.1384
14	Base Diameter	$d_b$	$d \cos \alpha_t$	44.6553	76.6445
15	Working Pitch Diameter	$d_{w1}$	$2 a_x \frac{d_1}{d_1 + d_2}$	49.1155	85.2695
		$d_{w2}$	$2 a_x \frac{d_2}{d_1 + d_2}$		
16	Working Helix Angle	$\beta_w$	$\tan^{-1} \left( \frac{d_w}{d} \tan \beta \right)$	20.4706°	30.6319°
17	Shaft Angle	$\Sigma$	$\beta_{w1} + \beta_{w2}$ or $\beta_{w1} - \beta_{w2}$	51.1025°	
18	Addendum	$h_{a1}$	$(1 + y - x_{n2}) m_n$	4.0793	3.4793
		$h_{a2}$	$(1 + y - x_{n1}) m_n$		
19	Whole Depth	$h$	$[2.25 + y - (x_{n1} + x_{n2})] m_n$	6.6293	
20	Outside Diameter	$d_a$	$d + 2 h_a$	56.0466	90.0970
21	Root Diameter	$d_f$	$d_a - 2 h$	42.7880	76.8384