

Forces acting on the driven gear can be calculated per **Equations (16-25)**.

$$\left. \begin{aligned} F_{u2} &= F_{a1} \sin \Sigma + F_{u1} \cos \Sigma \\ F_{a2} &= F_{u1} \sin \Sigma - F_{a1} \cos \Sigma \\ F_{r2} &= F_{r1} \end{aligned} \right\} \quad (16-25)$$

If the Σ term in **Equation (16-25)** is 90° , it becomes identical to **Equation (16-20)**. **Figure 16-16** presents the direction of forces in a screw gear mesh when the shaft angle $\Sigma = 90^\circ$ and $\beta_1 = \beta_2 = 45^\circ$.

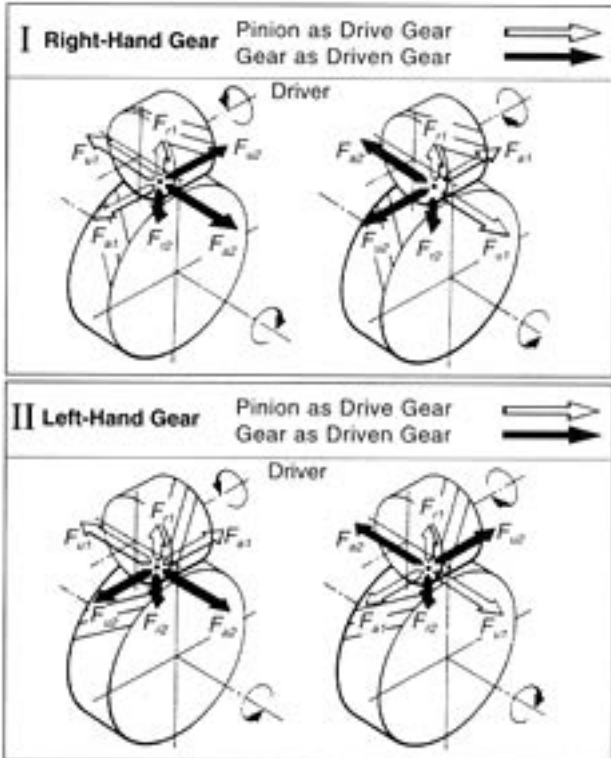


Fig. 16-16 Directions of Forces in a Screw Gear Mesh

SECTION 17 STRENGTH AND DURABILITY OF GEARS

The strength of gears is generally expressed in terms of bending strength and surface durability. These are independent criteria which can have differing criticalness, although usually both are important.

Discussions in this section are based upon equations published in the literature of the Japanese Gear Manufacturer Association (JGMA). Reference is made to the following JGMA specifications:

Specifications of JGMA:

JGMA 401-01	Bending Strength Formula of Spur Gears and Helical Gears
JGMA 402-01	Surface Durability Formula of Spur Gears and Helical Gears
JGMA 403-01	Bending Strength Formula of Bevel Gears
JGMA 404-01	Surface Durability Formula of Bevel Gears
JGMA 405-01	The Strength Formula of Worm Gears

Generally, bending strength and durability specifications are applied to spur and helical gears (including double helical and internal gears) used in industrial machines in the following range:

Module:	m	1.5 to 25 mm
Pitch Diameter:	d	25 to 3200 mm
Tangential Speed:	v	less than 25m/sec
Rotating Speed:	n	less than 3600 rpm

Conversion Formulas: Power, Torque and Force

Gear strength and durability relate to the power and forces to be transmitted. Thus, the equations that relate tangential force at the pitch circle, F_t (kgf), power, P (kw), and torque, T (kgf.m) are basic to the calculations. The relations are as follows:

$$F_t = \frac{102P}{v} = \frac{1.95 \times 10^6 P}{d_w n} = \frac{2000T}{d_w} \quad (17-1)$$

$$P = \frac{F_t v}{102} = \frac{10^{-6}}{19100} F_t d_w n = F_t d_w n \quad (17-2)$$

$$T = \frac{F_t d_w}{2000} = \frac{974P}{n} \quad (17-3)$$

where: v : Tangential Speed of Working Pitch Circle (m/sec)

$$v = \frac{d_w n}{19100}$$

d_w : Working Pitch Diameter (mm)

n : Rotating Speed (rpm)

17.1 Bending Strength Of Spur And Helical Gears

In order to confirm an acceptable safe bending strength, it is necessary to analyze the applied tangential force at the working pitch circle, F_t , vs. allowable force, F_{tlim} . This is stated as:

$$F_t < F_{tlim} \quad (17-4)$$

It should be noted that the greatest bending stress is at the root of the flank or base of the dedendum. Thus, it can be stated:

σ_F = actual stress on dedendum at root

σ_{Flim} = allowable stress

Then **Equation (17-4)** becomes **Equation (17-5)**

$$\sigma_F \leq \sigma_{Flim} \quad (17-5)$$

Equation (17-6) presents the calculation of F_{tlim} :

$$F_{tlim} = \sigma_{Flim} \frac{m_n b}{Y_F Y_c Y_B} \left(\frac{K_L K_{FX}}{K_V K_D} \right) \frac{1}{S_F} \quad (\text{kgf}) \quad (17-6)$$

Equation (17-6) can be converted into stress by **Equation (17-7)**:

$$\sigma_F = F_t \frac{Y_F Y_c Y_B}{m_n b} \left(\frac{K_V K_D}{K_L K_{FX}} \right) S_F \quad (\text{kgf/mm}^2) \quad (17-7)$$

17.1.1 Determination of Factors in the Bending Strength Equation

If the gears in a pair have different blank widths, let the wider one be b_w and the narrower one be b_s .

And if:

$b_w - b_s \leq mn$ b_w and b_s can be put directly into Equation (17-6).

$b_w - b_s > mn$ the wider one would be changed to $b_s + mn$ and the narrower one, b_s would be unchanged.

17.1.2 Tooth Profile Factor, Y_F

The factor Y_F is obtainable from **Figure 17-1** based on the equivalent number of teeth, Z_V and coefficient of profile shift, x , if the gear has a standard tooth profile with 20° pressure angle, per JIS B 1701. The theoretical limit of undercut is shown. Also, for profile shifted gears the limit of too narrow (sharp) a tooth top land is given. For internal gears, obtain the factor by considering the equivalent racks.

17.1.3 Load Distribution Factor, Y_ϵ

Load distribution factor is the reciprocal of radial contact ratio.

$$Y_\epsilon = \frac{1}{\epsilon_\alpha} \quad (17-8)$$

Table 17-1 shows the radial contact ratio of a standard spur gear.

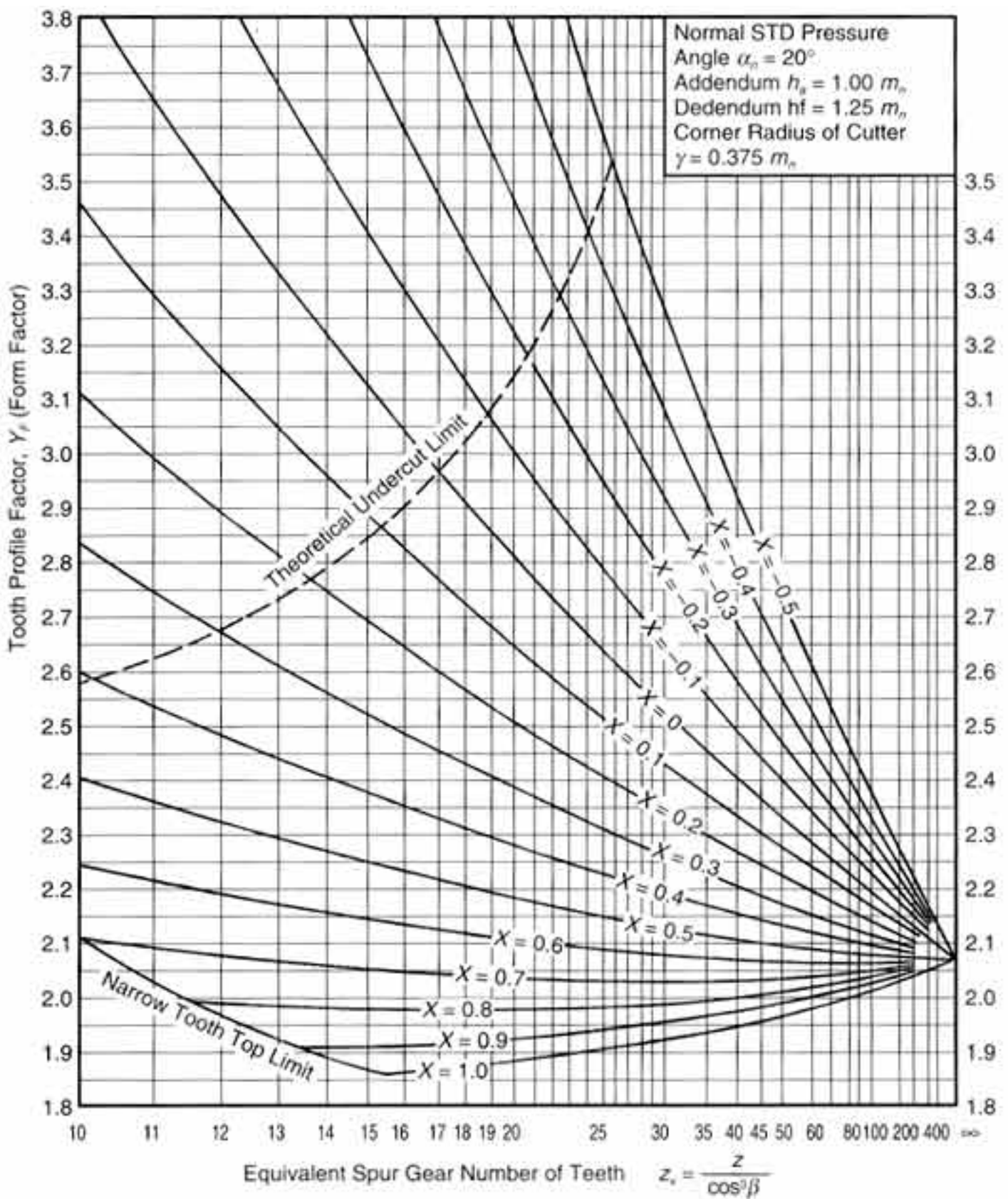


Fig. 17-1 Chart of Tooth Profile Factor, Y_f

Table 17-1 Radial Contact Ratio of Standard Spur Gears, ϵ_a ($\alpha = 20^\circ$)

	12	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	110	120	
12	1.420																					
15	1.451	1.481																				
20	1.489	1.519	1.557																			
25	1.516	1.547	1.584	1.612																		
30	1.537	1.567	1.605	1.633	1.654																	
35	1.553	1.584	1.622	1.649	1.670	1.687																
40	1.567	1.597	1.635	1.663	1.684	1.700	1.714															
45	1.578	1.609	1.646	1.674	1.695	1.711	1.725	1.736														
50	1.588	1.618	1.656	1.683	1.704	1.721	1.734	1.745	1.755													
55	1.596	1.626	1.664	1.691	1.712	1.729	1.742	1.753	1.763	1.771												
60	1.603	1.633	1.671	1.698	1.719	1.736	1.749	1.760	1.770	1.778	1.785											
65	1.609	1.639	1.677	1.704	1.725	1.742	1.755	1.766	1.776	1.784	1.791	1.797										
70	1.614	1.645	1.682	1.710	1.731	1.747	1.761	1.772	1.781	1.789	1.796	1.802	1.808									
75	1.619	1.649	1.687	1.714	1.735	1.752	1.765	1.777	1.786	1.794	1.801	1.807	1.812	1.817								
80	1.623	1.654	1.691	1.719	1.740	1.756	1.770	1.781	1.790	1.798	1.805	1.811	1.817	1.821	1.826							
85	1.627	1.657	1.695	1.723	1.743	1.760	1.773	1.785	1.794	1.802	1.809	1.815	1.821	1.825	1.830	1.833						
90	1.630	1.661	1.699	1.726	1.747	1.764	1.777	1.788	1.798	1.806	1.813	1.819	1.824	1.829	1.833	1.837	1.840					
95	1.634	1.664	1.702	1.729	1.750	1.767	1.780	1.791	1.801	1.809	1.816	1.822	1.827	1.832	1.836	1.840	1.844	1.847				
100	1.636	1.667	1.705	1.732	1.753	1.770	1.783	1.794	1.804	1.812	1.819	1.825	1.830	1.835	1.839	1.843	1.846	1.850	1.853			
110	1.642	1.672	1.710	1.737	1.758	1.775	1.788	1.799	1.809	1.817	1.824	1.830	1.835	1.840	1.844	1.848	1.852	1.855	1.858	1.863		
120	1.646	1.676	1.714	1.742	1.762	1.779	1.792	1.804	1.813	1.821	1.828	1.834	1.840	1.844	1.849	1.852	1.856	1.859	1.862	1.867	1.871	
RACK	1.701	1.731	1.769	1.797	1.817	1.834	1.847	1.859	1.868	1.876	1.883	1.889	1.894	1.899	1.903	1.907	1.911	1.914	1.917	1.926		

17.1.4 Helix Angle Factor, Y_β

Helix angle factor can be obtained from **Equation(17-9)**.

$$\left. \begin{aligned} \text{When } 0 \leq \beta \leq 30^\circ, \text{ then } Y_\beta &= 1 - \frac{\beta}{120} \\ \text{When } \beta > 30^\circ, \text{ then } Y_\beta &= 0.75 \end{aligned} \right\} (17-9)$$

17.1.5 Life Factor, K_L

We can choose the proper life factor, K_L from **Table 17-2**.

The number of cyclic repetitions means the total loaded meshings during its lifetime.

17.1.6 Dimension Factor of Root Stress, K_{FX}

Generally, this factor is unity.

$$K_{FX} = 1.00 \quad (17-10)$$

17.1.7 Dynamic Load Factor, K_V

Dynamic load factor can be obtained from **Table 17-3** based on the precision of the gear and its pitch line linear speed.

Table 17-2 Life Factor, K_L

Number of Cyclic Repetitions	Hardness (1) HB 120 ... 220	Hardness(2) Over HB 220	Gears with Carburizing Gears with Nitriding
Under 10000	1.4	1.5	1.5
Approx. 10^5	1.2	1.4	1.5
Approx. 10^6	1.1	1.1	1.1
Approx. 10^7	1.0	1.0	1.0

NOTES: (1) Cast iron gears apply to this column.

(2) For induction hardened gears, use the core hardness.

Table 17-3 Dynamic Load Factor, K_V

Precision Grade of Gears form JIS B1702		Tangential Speed at Pitch Line (m/s)						
		Under 1	1 to less than 3	3 to less than 5	5 to less than 8	8 to less than 18	12 to less than 18	18 to less than 25
Tooth Profile		Under 1	1 to less than 3	3 to less than 5	5 to less than 8	8 to less than 18	12 to less than 18	18 to less than 25
Unmodified	Modified							
	1	-	-	1.0	1.0	1.1	1.2	1.3
1	2	-	1.0	1.05	1.1	1.2	1.3	1.5
2	3	1.0	1.1	1.15	1.2	1.3	1.5	
3	4	1.0	1.2	1.3	1.4	1.5		
4	-	1.0	1.3	1.4	1.5			
5	-	1.1	1.4	1.5				
6	-	1.2	1.5					

Table 17-4 Overload Factor, K_O

Impact from Prime Mover	Impact from Load Side of Machine		
	Uniform Load	Medium Impact load	Heavy Impact load
Uniform Load (Motor, Turbine, Hydraulic Motor)	1.0	1.25	1.75
Light Impact Load (Multicylinder Engine)	1.25	1.5	2.0
Medium Impact Load (Single Cylinder Engine)	1.5	1.75	2.25

17.1.8 Overload Factor, K_O

Overload factor, K_O , is the quotient of actual tangential force divided by nominal tangential force, F_T . If tangential force is unknown, **Table 17-4** provides guiding values.

$$K_O = \frac{\text{Actual tangential force}}{\text{Nominal tangential force, } F_1} \quad (17-11)$$

17.1.9 Safety Factor for Bending Failure, S_F

Safety factor, S_F , is too complicated to be decided precisely. Usually, it is set to at least 1.2.

17.1.10 Allowable Bending Stress at Root, σ_{Flim}

For the unidirectionally loaded gear, the allowable bending stresses at the root are shown in **Tables 17-5 to 17-8**. In these tables, the value of $a_{F,m}$ is the quotient of the tensile fatigue limit divided by the stress concentration factor 1.4. If the load is bidirectional, and both sides of the tooth are equally loaded, the value of allowable bending stress should be taken as 2/3 of the given value in the table. The core hardness means hardness at the center region of the root.

See **Table 17-5** for s_{Flim} of gears without case hardening. **Table 17-6** gives s_{Flim} of gears that are induction hardened; and **Tables 17-7** and **17-8** give the values for carburized and nitrided

gears, respectively. In **Tables 17-8A** and **17-8B**, examples of calculations are given.

Table 17-5 Gears Without Case Hardening

Material	Arrow Indicate the ranges	Hardness		Tensile Strength Lower limit kgf/mm ² (Reference)	σ_{Flim} kgf/mm ²
		HB	HV		
Cast Steel Gear	SC37 SC42 SC46 SC49 SCC3			37	10.4
				42	12.0
				46	13.2
				49	14.2
				55	15.8
				60	17.2
Normalization Carbon Steel Gear		120	126	39	13.8
		130	136	42	14.8
		140	147	45	15.8
		150	157	48	16.8
		160	167	51	17.6
		170	178	55	18.4
		180	189	58	19.0
		190	200	61	19.5
		200	210	64	20
		210	221	68	20.5
		220	231	71	21
		230	242	74	21.5
		240	252	77	22
		250	263	81	22.5
Quenched and Tempered Carbon Steel Gear		160	167	51	18.2
		170	178	55	19.4
		180	189	58	20.2
		190	200	61	21
		200	210	64	22
		210	221	68	23
		220	231	71	23.5
		230	242	74	24
		240	252	77	24.5
		250	263	81	25
		260	273	84	25.5
		270	284	87	26
		280	295	90	26
		290	305	93	26.5
Quenched and Tempered Alloy Steel Gear		220	231	71	25
		230	242	74	26
		240	252	77	27.5
		250	263	81	28.5
		260	273	84	29.5
		270	284	87	31
		280	295	90	32
		290	305	93	33
		300	316	97	34
		310	327	100	35
		320	337	103	36.5
		330	347	106	37.5
		340	358	110	39
		350	369	113	40
360	380	117	41		

Table 17-6 Induction Hardened Gears

Material	Arrow indicate the range	Heat Treatment before Induction Hardening	Core Hardness		Surface Hardness HV	σ_{Flim} Kgf/mm ²
			HB	HV		
Structural Carbon Steel Hardened Throughout	S48C ↔ S43C	Normalized	160	167	More than 550	21
			180	189	"	21
			220	231	"	21.5
			240	252	"	22
	S48C ↔ S43C	Quenched and Tempered	200	210	More than 550	23
			210	221	"	23.5
			220	231	"	24
			230	242	"	24.5
			240	252	"	25
			250	263	"	25
Structural Alloy Steel Hardened Throughout	SCM440 ↔ SMn443 ↔ SNC836 ↔ SCM435 SNCM439	Quenched and Tempered	230	242	More than 550	27
			240	252	"	28
			250	263	"	29
			260	273	"	30
			270	284	"	31
			280	295	"	32
			290	305	"	33
			300	316	"	34
			310	327	"	35
			320	337	"	36.5
Hardened Except Root Area						75% of the above

NOTES: 1. if a gear is not quenched completely, or not evenly, or has quenching cracks, the σ_{Flim} will drop dramatically.
 2. If the hardness after quenching is relatively low, the value of σ_{Flim} should be that given in **Table 17-5**.

Table 17-7 Carburized Gears

Material	Arrows indicate the ranges	core hardness		σ_{Flim} Kgf/mm ²
		HB	HV	
Structural carbon Steel	S15C S15CK	140	147	18.2
		150	157	19.6
		160	167	21
		170	178	22
		180	189	23
		190	200	24
Structural Alloy Steel	SCM415 ↔ SNC415 SCM420 ↔ SNC815 SNCM420	220	231	34
		230	242	36
		240	252	38
		250	263	39
		260	273	41
		270	284	42.5
		280	295	44
		290	305	45
		300	316	46
		310	327	47
		320	337	48
		330	347	49
		340	358	50
		350	369	51
360	380	51.5		
370	390	52		

Table 17-8 Nitrided Gears

Material	Surface Hardness (Reference)	Core Hardness		σ_{Flim} kgf/mm ²
Alloy Steel except Nitriding Steel	More than HV 650	220	231	30
		240	252	33
		260	273	36
		280	295	38
		300	316	40
		320	337	42
		340	358	44
		360	380	46
Nitriding Steel SACM645	More than HV 650	220	231	32
		240	252	35
		260	273	38
		280	295	41
		300	316	44

NOTE: The above two tables apply only to those gears which have adequate depth of surface hardness. Otherwise, the gears should be rated according to **Table 17-5**.