Table 15-7 Allowable Values of Running Errors, mm

Grade	Tooth-to-Tooth Composite Error	Total Composite Error
0	1.12m + 3.55	(1.4W + 4.0) + 0.5(1.12m + 3.55)
1	1.6m + 5.0	(2.0W + 5.6) + 0.5(1.6m + 5.0)
2	2.24m + 7.1	(2.8W + 8.0) + 0.5(2.24m + 7.1)
3	3.15m + 10.0	(4.0W + 11.2) + 0.5(3.15m + 10.0)
4	4.5m + 14.0	(5.6W + 16.0) + 0.5(4.5m + 14.0)
5	6.3m + 20.0	(8.0W + 22.4) + 0.5(6.3m + 20.0)
6	9.0m + 28.0	(11.2W + 31.5) + 0.5(9.0m + 28.0)
7	12.5m + 40.0	(22.4W + 63.0) + 0.5(12.5m + 40.0)
8	18.0m + 56.0	(45.0W + 125.0) + 0.5(18.0m + 56.0)

where: W = Tolerance unit = $\sqrt[3]{d} + 0.65m$ (μm)
d = Pitch diameter (mm)
m = Module

SECTION 16 GEAR FORCES

In designing a gear, it is important to analyze the magnitude and direction of the forces acting upon the gear teeth, shaft, bearings, etc. In analyzing these forces, an idealized assumption is made that the tooth forces are acting upon the central part of the tooth flank.

16.1 Forces In A Spur Gear Mesh

The spur gear's transmission force F_n, which is normal to the tooth surface, as in **Figure 16-1**, can be resolved into a tangential component, F_u, and a radial component, F_r Refer to **Equation (16-1)**.

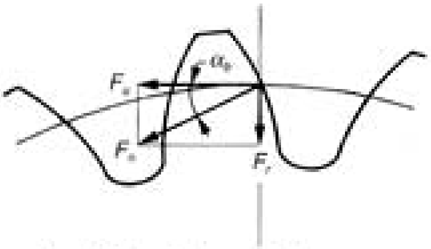

Fig. 16-1 Forces Acting on a Spur Gear Mesh

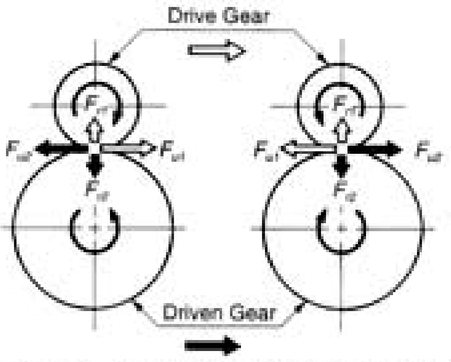

Fig. 16-2 Directions of Forces Acting on a Spur Gear Mesh

$$\left.\begin{aligned} F_u &= F_n \cos\alpha_b \\ F_r &= F_n \sin\alpha_b \end{aligned}\right\} \quad (16\text{-}1)$$

The direction of the forces acting on the gears are shown in **Figure 16-2**. The tangential component of the drive gear, F_{u1}, is equal to the driven gear's tangential component, F_{u2}, but the directions are opposite. Similarly, the same is true of the radial components.

16.2 Forces In A Helical Gear Mesh

The helical gear's transmission force, F_n, which is normal to the tooth surfaces, can be resolved into a tangential component, F_1, and a radial component, F_r, as shown in **Figure 16-3**.

Table 16-1 Forces Acting Upon a Gear

Types of Gears		Tangential Force, F_u	Axial Force, F_a	Radial Force, f_r
Spur Gear		$F_u = \frac{2000\,T}{d}$	———	$F_u \tan\alpha$
Helical Gear			$F_u \tan\beta$	$F_u \frac{\tan\alpha_n}{\cos\beta}$
Straight Bevel Gear			When convex surface is working:	
Spiral Bevel Gear		$F_u = \frac{2000\,T}{d_m}$ d_m is the central pitch diameter $d_m = d - b\sin\delta$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$
			When concave surface is working:	
			$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta)$	$\frac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta - \sin\beta_m\sin\delta)$
Worm drive	Worm (Driver)	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$	$F_u \frac{\sin\alpha_n}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$
	Wheel (Driven)	$F_u \frac{\cos\alpha_n\cos\gamma - \mu\sin\gamma}{\cos\alpha_n\sin\gamma + \mu\cos\gamma}$	F_u	
Screw Gear (Σ = 90º) β=45º	Driver gear	$F_u = \frac{2000\,T_1}{d_1}$	$F_u \frac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	$F_u \frac{\sin\alpha_n}{\cos\alpha_n\cos\beta + \mu\sin\beta}$
	Driven Gear	$F_u \frac{\cos\alpha_n\sin\beta - \mu\cos\beta}{\cos\alpha_n\cos\beta + \mu\sin\beta}$	F_u	

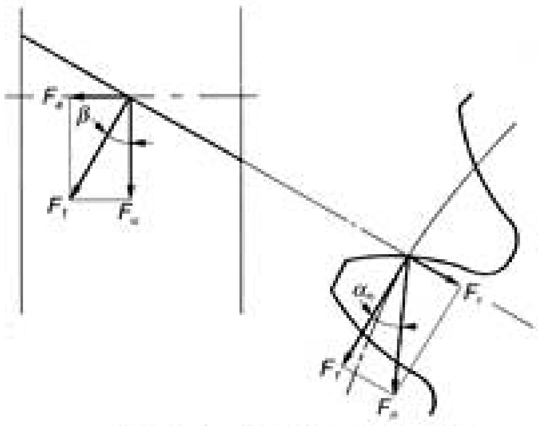

Fig. 16-3 Forces Acting on a Helical Gear Mesh

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_r &= F_n \sin\alpha_n \end{aligned}\right\} \quad (\textbf{16-2})$$

The tangential component, F_1, can be further resolved into circular subcomponent, F_u, and axial thrust subcomponent, F_a.

$$\left.\begin{aligned} F_u &= F_1 \cos\beta \\ F_a &= F_1 \sin\beta \end{aligned}\right\} \quad (\textbf{16-3})$$

Substituting and manipulating the above equations result in:

$$\left.\begin{aligned} F_a &= F_u \tan\beta \\ F_r &= F_u \frac{\tan\alpha_n}{\cos\beta} \end{aligned}\right\} \quad (\textbf{16-4})$$

The directions of forces acting on a helical gear mesh are shown in **Figure 16-4**. The axial thrust sub-component from drive gear, F_{a1}, equals the driven gear's, F_{a2}, but their directions are opposite. Again, this case is the same as tangential components F_{u1}, F_{u2} and radial components F_{r1}, F_{r2}.

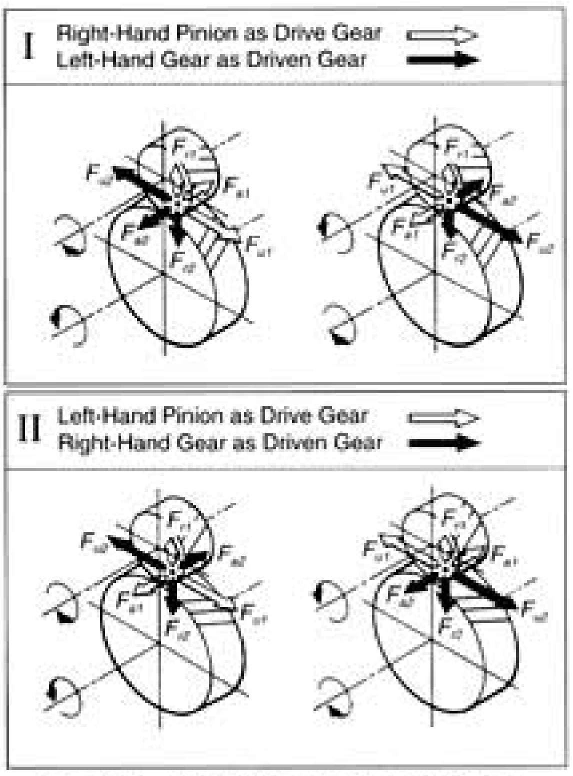

Fig. 16-4 Directions of Forces Acting on a Helical Gear Mesh

16.3 Forces In A Straight Bevel Gear Mesh

The forces acting on a straight bevel gear are shown in **Figure 16-5**. The force which is normal to the central part of the tooth face, F_n, can be split into tangential component, F_u, and radial component, F_1, in the normal plane of the tooth.

$$\left.\begin{aligned} F_u &= F_n \cos\alpha \\ F_1 &= F_n \sin\alpha \end{aligned}\right\} \quad (\textbf{16-5})$$

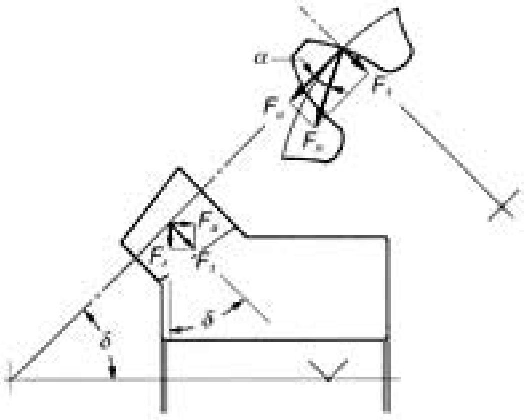

Fig. 16-5 Forces Acting on a Straight Bevel Gear Mesh

Again, the radial component, F_{r1}, can be divided into an axial force, F_a, and a radial force, F_r, perpendicular to the axis.

$$\left.\begin{aligned} F_a &= F_1 \sin\delta \\ F_r &= F_1 \cos\delta \end{aligned}\right\} \quad (\textbf{16-6})$$

And the following can be derived:

$$\left.\begin{aligned} F_a &= F_u \tan\alpha_n \sin\delta \\ F_r &= F_u \tan\alpha_n \cos\delta \end{aligned}\right\} \quad (\textbf{16-7})$$

Let a pair of straight bevel gears with a shaft angle $\Sigma = 90^\circ$, a pressure angle $\alpha_n = 20^\circ$ and tangential force, F_u, to the central part of tooth face be 100. Axial force, F_a, and radial force, F_r, will be as presented in **Table 16-2**.

Table 16-2 Values of Axial Force, F_a, and Radial Force, F_r

(1) Pinion

Force on the Gear tooth	Ratio of Number of Teeth $\frac{Z_2}{Z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force	25.7	20.2	16.3	13.5	11.5	8.8	7.1
Radial Force	25.7	30.3	32.6	33.8	34.5	35.3	35.7

(2) Gear

Force on the Gear tooth	Ratio of Number of Teeth $\frac{Z_2}{Z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Axial Force	25.7	30.3	32.6	33.8	34.5	35.3	35.7
Radial Force	25.7	20.2	16.3	13.5	11.5	8.8	7.1

Figure 16-6 contains the directions of forces acting on a straight bevel gear mesh. In the meshing of a pair of straight bevel gears with shaft angle Σ = 90º, all the forces have relations as per **Equations (16-8)**

$$\left.\begin{aligned} F_{u1} &= F_{u2} \\ F_{r1} &= F_{a2} \\ F_{a1} &= F_{r2} \end{aligned}\right\} \qquad (\mathbf{16\text{-}8})$$

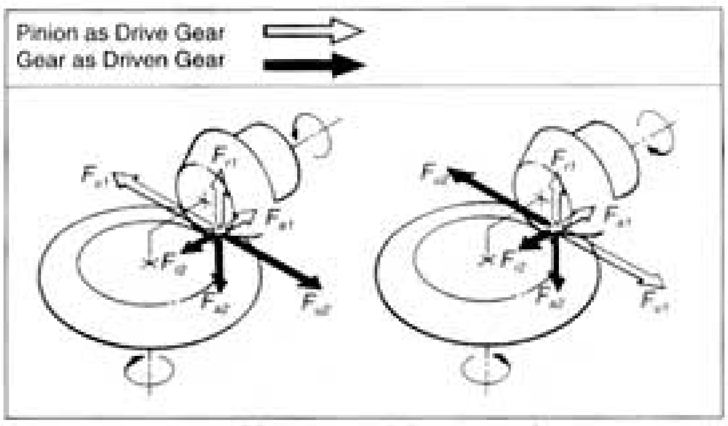

Fig. 16-6 Directions of Forces Acting on a Straight Bevel Gear Mesh

16.4 Forces In A Spiral Bevel Gear Mesh

Spiral gear teeth have convex and concave sides. Depending on which surface the force is acting on, the direction and magnitude changes. They differ depending upon which is the driver and which is the driven. **Figure 16-7** presents the profile orientations of right- and left-hand spiral teeth. If the profile of the driving gear is convex, then the profile of the driven gear must be concave. **Table 16-3** presents the concave/convex relationships.

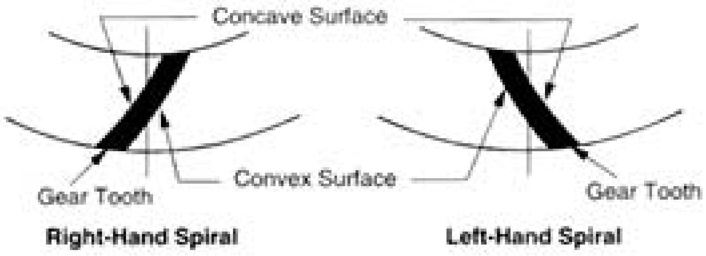

Fig. 16-7 Convex Surface and Concave Surface of a Spiral Bevel Gear

Table 16-3 Concave and Convex Sides of a Spiral Bevel Gear Mesh

Right-Hand Gear as Drive Gear

Rotational Direction of Drive Gear		
	Right-Hand Drive Gear	**left Hand Driven Gear**
Clockwise	Convex	Concave
Counterclockwise	Concave	Convex

Left-Hand Gear as Drive Gear

Rotational Direction of Drive Gear	**Meshing Tooth Face**	
	left Hand Driven Gear	**Right-Hand Drive Gear**
Clockwise	Concave	Convex
Counterclockwise	Convex	Concave

NOTE: The rotational direction of a bevel gear is defined as the direction one sees viewed along the axis from the back cone to the apex.

16.4.1 Tooth Forces on a Convex Side Profile

The transmission force, F_n can be resolved into components F_1, and F_t as (see **Figure 16-8**):

$$\left.\begin{aligned} F_1 &= F_n\cos\alpha_n \\ F_t &= F_n\sin\alpha_n \end{aligned}\right\} \qquad (\mathbf{16\text{-}9})$$

Then F_1 can be resolved into components F_u and F_s:

$$\left.\begin{aligned} F_u &= F_1\cos\beta_m \\ F_s &= F_1\sin\beta_m \end{aligned}\right\} \qquad (\mathbf{16\text{-}10})$$

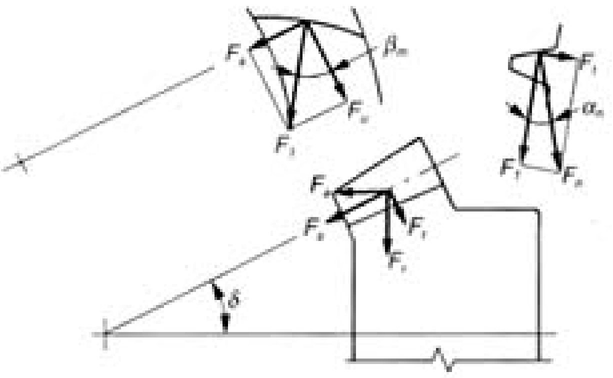

Fig. 16-8 When Meshing on the Convex Side of Tooth Face

On the axial surface, F_t, and F_s can be resolved into axial and radial subcomponents.

$$\left.\begin{aligned} F_a &= F_t\sin\delta - F_s\cos\delta \\ F_r &= F_t\cos\delta + F_s\sin\delta \end{aligned}\right\} \qquad (\mathbf{16\text{-}11})$$

By substitution and manipulation. we obtain:

$$\left.\begin{aligned} F_a &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n\sin\delta - \sin\beta_m\cos\delta) \\ F_r &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n\cos\delta + \sin\beta_m\sin\delta) \end{aligned}\right\} \qquad (\mathbf{16\text{-}12})$$

16.4.2 Tooth Forces on a Concave Side Profile

On the surface which is normal to the tooth profile at the central portion of the tooth, the transmission force, F_n can be split into F_1 and F_t, as (see **Figure 16-9**):

$$\left.\begin{aligned} F_1 &= F_n\cos\alpha_n \\ F_t &= F_n\sin\alpha_n \end{aligned}\right\} \qquad (\mathbf{16\text{-}13})$$

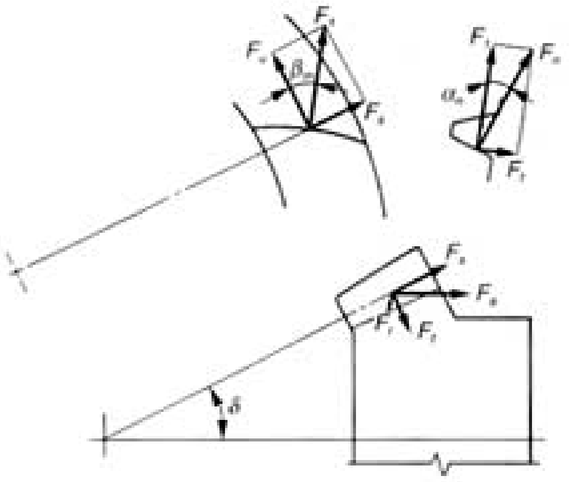

Fig. 16-9 When Meshing on the Concave Side of Tooth Face

And F_1 can be separated into components F_u and F_s on the pitch surface:

$$\left.\begin{aligned} F_u &= F_1\cos\beta_m \\ F_s &= F_1\sin\beta_m \end{aligned}\right\} \qquad (\mathbf{16\text{-}14})$$

So far, the equations are identical to the convex case. However, differences exist in the signs for equation terms. On the axial surface, F_1 and F_s can be resolved into axial and radial subcomponents. Note the sign differences.

$$\left.\begin{aligned} F_a &= F_t \sin\delta + F_s \cos\delta \\ F_r &= F_t \cos\delta - F_s \sin\delta \end{aligned}\right\} \qquad (\mathbf{16\text{-}15})$$

The above can be manipulated to yield:

$$\left.\begin{aligned} F_a &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \sin\delta + \sin\beta_m \cos\delta) \\ F_r &= \frac{F_u}{\cos\beta_m}(\tan\alpha_n \cos\delta - \sin\beta_m \sin\delta) \end{aligned}\right\} \qquad (\mathbf{16\text{-}16})$$

Let a pair of spiral bevel gears have a shaft angle $\Sigma = 90°$, a pressure angle $\alpha_n = 20°$, and a spiral angle $\beta_m = 35°$. If the tangential force, F_u to the central portion of the tooth face is 100, the axial thrust force, F_a, and radial force, F_r have the relationship shown in **Table 16-4**.

The value of axial force, F_a, of a spiral bevel gear, from **Table 16-4**, could become negative. At that point, there are forces tending to push the two gears together. If there is any axial play in the bearing, it may lead to the undesirable condition of the mesh having no backlash. Therefore, it is important to pay particular attention to axial plays. From **Table 16-4(2)**, we understand that axial thrust force, F_a changes from positive to negative in the range of teeth ratio from 1.5 to 2.0 when a gear carries force on the convex side. The precise turning point of axial thrust force, F_a, is at the teeth ratio $z_1 / z_2 = 1.57357$.

Table 16-4 Values of Axial Thrust Force, F_a and Radial Force, F_r

(1) Pinion

Meshing Tooth Face	Ratio of Number of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	80.9	82.9	82.5	81.5	80.5	78.7	77.4
	-18.1	-1.9	8.4	15.2	20.0	26.1	29.8
Convex Side of Tooth	-18.1	-33.6	-42.8	-48.5	-52.4	-57.2	-59.9
	80.9	75.8	71.1	67.3	64.3	60.1	57.3

(2) Gear

Meshing Tooth Face	Ratio of Number of Teeth $\frac{z_2}{z_1}$						
	1.0	1.5	2.0	2.5	3.0	4.0	5.0
Concave Side of Tooth	80.9	75.8	71.1	67.3	64.3	60.1	57.3
	-18.1	-33.6	-42.8	-48.5	-52.4	-57.2	-59.9
Convex Side of Tooth	-18.1	-1.9	8.4	15.2	20.0	26.1	29.8
	80.9	82.9	82.5	81.5	80.5	78.7	77.4

Figure 16-10 describes the forces for a pair of spiral bevel gears with shaft angle $\Sigma = 90°$, pressure angle $\alpha_n = 20°$, spiral angle $\beta_m = 35°$ and the teeth ratio, u, ranging from 1 to 1 .57357.

Figure 16-11 expresses the forces of another pair of spiral bevel gears taken with the teeth ratio equal to or larger than 1 .57357.

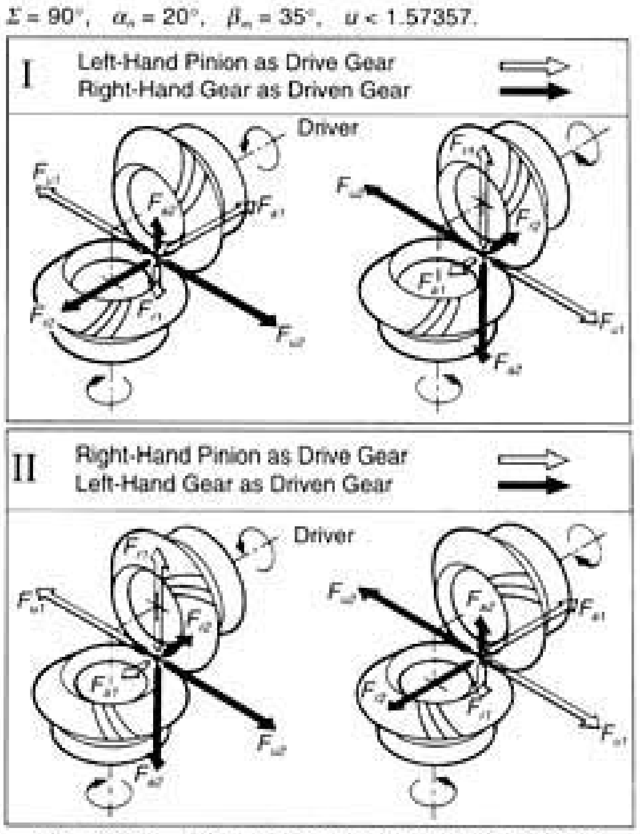

Fig. 16-10 The Direction of Forces Carried by Spiral Bevel Gears (1)

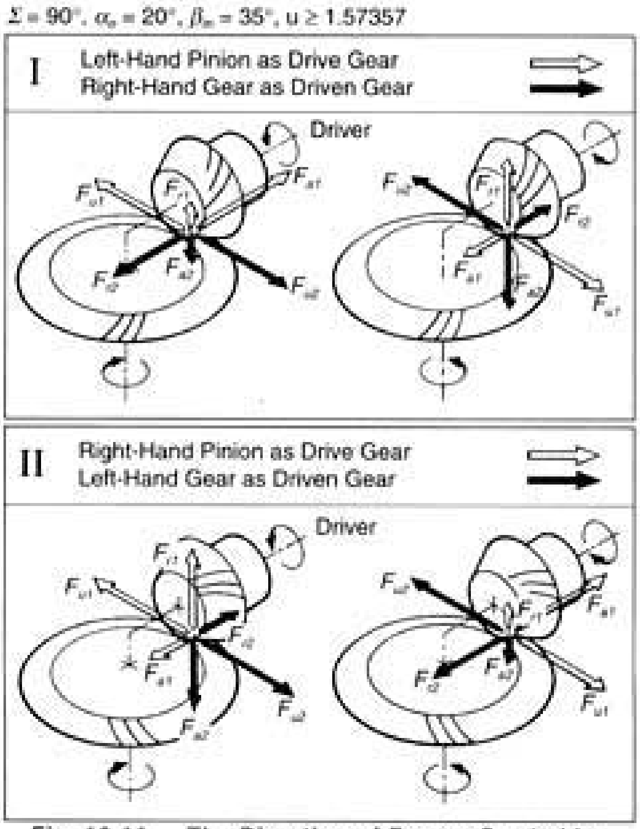

Fig. 16-11 The Direction of Forces Carried by Spiral Bevel Gears (2)

16.5 Forces In A Worm Gear Mesh

16.5.1 Worm as the Driver

For the case of a worm as the driver, **Figure 16-12**, the transmission force, F_n which is normal to the tooth surface at the pitch circle can be resolved into components F_1 and F_{r1}

$$\left.\begin{aligned} F_1 &= F_n \cos\alpha_n \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \quad (16\text{-}17)$$

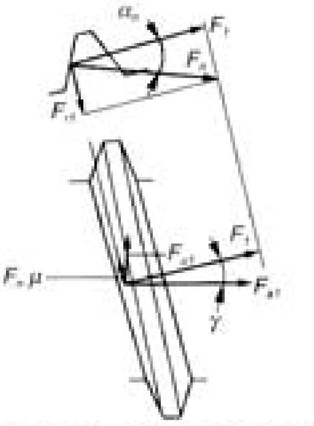

Fig. 16-12 Forces Acting on the Tooth Surface of a Worm

At the pitch surface of the worm, there is, in addition to the tangential component, F_1, a friction sliding force on the tooth surface, mF_n These two forces can be resolved into the circular and axial directions as:

$$\left.\begin{aligned} F_{u1} &= F_1 \sin\gamma + F_n \mu\cos\gamma \\ F_{a1} &= F_1 \cos\gamma - F_n \mu\sin\gamma \end{aligned}\right\} \quad (16\text{-}18)$$

and by substitution, the result is:

$$\left.\begin{aligned} F_{u1} &= F_n (\cos\alpha_n \sin\gamma + \mu\cos\gamma) \\ F_{a1} &= F_n (\cos\alpha_n \cos\gamma - \mu\sin\gamma) \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \quad (16\text{-}19)$$

Figure 16-13 presents the direction of forces in a worm gear mesh with a shaft angle $\Sigma = 90º$. These forces relate as follows:

$$\left.\begin{aligned} F_{a1} &= F_{u2} \\ F_{u1} &= F_{a2} \\ F_{r1} &= F_{r2} \end{aligned}\right\} \quad (16\text{-}20)$$

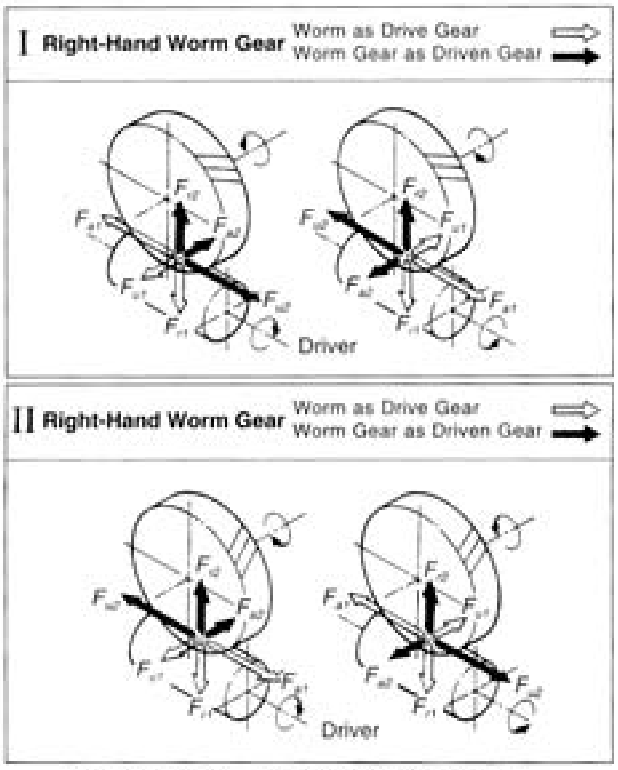

Figure 16-13 Direction of Forces in a Worm Gear Mesh

The coefficient of friction has a great effect on the transmission of a worm gear. **Equation (16-21)** presents the efficiency when the worm is the driver.

$$\eta_R = \frac{T_2}{T_1 i} = \frac{F_{u2}}{F_{u1}} \tan\gamma = \frac{\cos\alpha_n \cos\gamma - \mu\sin\gamma}{\cos\alpha_n \sin\gamma + \mu\cos\gamma} \tan\gamma \quad (16\text{-}21)$$

16.5.2 Worm Gear as the Driver

For the case of a worm gear as the driver, the forces are as in **Figure 16-14** and per **Equations (16-22)**.

$$\left.\begin{aligned} F_{u2} &= F_n(\cos\alpha_n \cos\gamma + \mu\sin\gamma) \\ F_{a2} &= F_n(\cos\alpha_n \sin\gamma - \mu\cos\gamma) \\ F_{r2} &= F_n \sin\alpha_n \end{aligned}\right\} \quad (16\text{-}22)$$

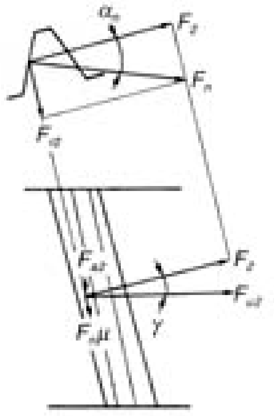

Fig. 16-14 Forces in a Worm Gear Mesh

When the worm and worm gear are at 90º shaft angle, **Equations (16-20)** apply.Then, when the worm gear is the driver, the transmission efficiency n1 is expressed as per **Equation (16-23)**.

$$\eta_I = \frac{T_1 i}{T_2} = \frac{F_{u1}}{F_{u2}\tan\gamma} = \frac{\cos\alpha_n \sin\gamma - \mu\cos\gamma}{\cos\alpha_n \cos\gamma + \mu\sin\gamma} \frac{1}{\tan\gamma} \quad (16\text{-}23)$$

The equations concerning worm and worm gear forces contain the coefficient μ. This indicates the coefficient of friction is very important in the transmission of power.

16.6 Forces In A Screw Gear Mesh

The forces in a screw gear mesh are similar to those in a worm gear mesh. For screw gears that have a shaft angle $\Sigma = 90º$, merely replace the worms lead angle γ, in **Equation (16-22)**, with the screw gear's helix angle b_1

In the general case when the shaft angle is not 90º, as in **Figure 16-15**, the driver screw gear has the same forces as for a worm mesh. These are expressed in **Equations (16-24)**.

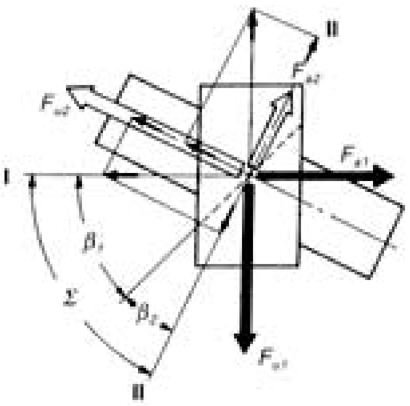

Fig. 16-15 The Forces in a Screw Gear Mesh

$$\left.\begin{aligned} F_{u1} &= F_n(\cos\alpha_n \cos\beta_1 + \mu\sin\beta_1) \\ F_{a1} &= F_n(\cos\alpha_n \sin\beta_1 - \mu\cos\beta_1) \\ F_{r1} &= F_n \sin\alpha_n \end{aligned}\right\} \quad (19\text{-}24)$$