Table 13-1 Equations of Speed Ratio for a Planetary Type

No.	Description	Sun Gear A Z_a	Planet Gear B Z_b	Internal Gear C Z_c	Carrier D
1	Rotate sun gear A once while holding carrier	+ 1	$-\frac{Z_a}{Z_b}$	$-\frac{Z_a}{Z_c}$	0
2	System is fixed as a whole while rotating $+(Z_a/Z_c)$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}$
3	Sum of 1 and 2	$1+\frac{Z_a}{Z_c}$	$+\frac{Z_a}{Z_c}-\frac{Z_a}{Z_b}$	0 (fixed)	$+\frac{Z_a}{Z_c}$

Table 13-2 Equations of speed Ratio for a Solar Type

No.	Description	Sun Gear A Z_a	Planet Gear B Z_b	Internal Gear C Z_c	Carrier D
1	Rotate sun gear A once while holding carrier	+ 1	$-\frac{Z_a}{Z_b}$	$-\frac{Z_a}{Z_c}$	0
2	System is fixed as a whole while rotating $+(Z_a/Z_c)$	- 1	- 1	- 1	- 1
3	Sum of 1 and 2	0 (fixed)	$-\frac{Z_a}{Z_b}-1$	$-\frac{Z_a}{Z_c}-1$	- 1

SECTION 14 BACKLASH

Up to this point the discussion has implied that there is no backlash. If the gears are of standard tooth proportion design and operate on standard center distance they would function ideally with neither backlash nor jamming.

Backlash is provided for a variety of reasons and cannot be designated without consideration of machining conditions. The general purpose of backlash is to prevent gears from jamming by making contact on both sides of their teeth simultaneously. A small amount of backlash is also desirable to provide for lubricant space and differential expansion between the gear components and the housing. Any error in machining which tends to increase the possibility of jamming makes it necessary to increase the amount of backlash by at least as much as the possible cumulative errors. Consequently, the smaller the amount of backlash, the more accurate must be the machining of the gears. Runout of both gears, errors in profile, pitch, tooth thickness, helix angle and center distance Ä all are factors to consider in the specification of the amount of backlash. On the other hand, excessive backlash is objectionable, particularly if the drive is frequently reversing or if there is an overrunning load. The amount of backlash must not be excessive for the requirements of the job, but it should be sufficient so that machining costs are not higher than necessary.

In order to obtain the amount of backlash desired, it is necessary to decrease tooth thickness. See **Figure 14-1**. This decrease must almost always be greater than the desired backlash because of the errors in manufacturing and assembling. Since the amount of the decrease in tooth thickness depends upon the accuracy of machining, the allowance for a specified backlash will vary according to the manufacturing conditions.

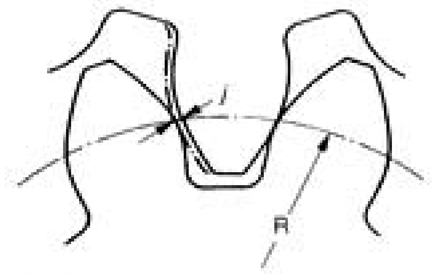


Figure 14-1 Backlash, j, Between Two Gears

It is customary to make half of the allowance for backlash on the tooth thickness of each gear of a pair, although there are exceptions. For example, on pinions having very low numbers of teeth, it is desirable to provide all of the allowance on the mating gear so as not to weaken the pinion teeth.

In spur and helical gearing, backlash allowance is usually obtained by sinking the hob deeper into the blank than the theoretically standard depth. Further, it is true that any increase or decrease in center distance of two gears in any mesh will cause an increase or decrease in backlash. Thus, this is an alternate way of designing backlash into the system.

In the following, we give the fundamental equations for the determination of backlash in a single gear mesh. For the determination of backlash in gear trains, it is necessary to sum the backlash of each mated gear pair. However, to obtain the total backlash for a series of meshes, it is necessary to take into account the gear ratio of each mesh relative to a chosen reference shaft in the gear train. For details, see Reference 10 at the end of the technical section.

14.1 Definition Of Backlash

Backlash is defined in **Figure 14-2(a)** as the excess thickness of tooth space over the thickness of the mating tooth. There are two basic ways in which backlash arises: tooth thickness is below the zero backlash value; and the operating center distance is greater than the zero backlash value.

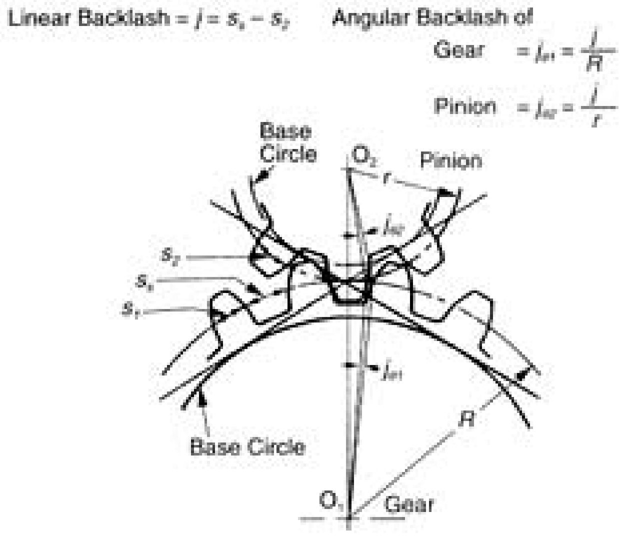


Fig. 14-2(a) Geometrical Definition of Angular Backlash

If the tooth thickness of either or both mating gears is less than the zero backlash value, the amount of backlash introduced in the mesh is simply this numerical difference:

$j = S_{std} - S_{act} = \Delta S$ (**14-1**)

where:

j = linear backlash measured along the pitch circle (**Figure 14-2(b)**)

S_{std} = no backlash tooth thickness on the operating pitch circle, which is the standard tooth thickness for ideal gears

S_{act} = actual tooth thickness

Backlash, Along Line-of-Action = $j_n = j\cos\alpha$

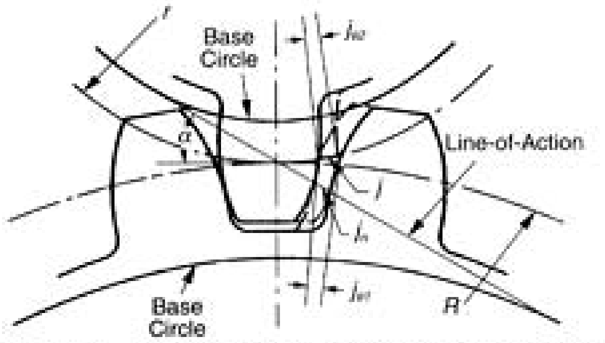


Fig. 14-2(b) Geometrical Definition of Linear Backlash

When the center distance is increased by a relatively small amount, Δa, a backlash space develops between mating teeth, as in **Figure 14-3**. The relationship between center distance increase and linear backlash j_n along the line-of-action is:

$j_n = 2\,\Delta a \sin\alpha$ (**14-2**)

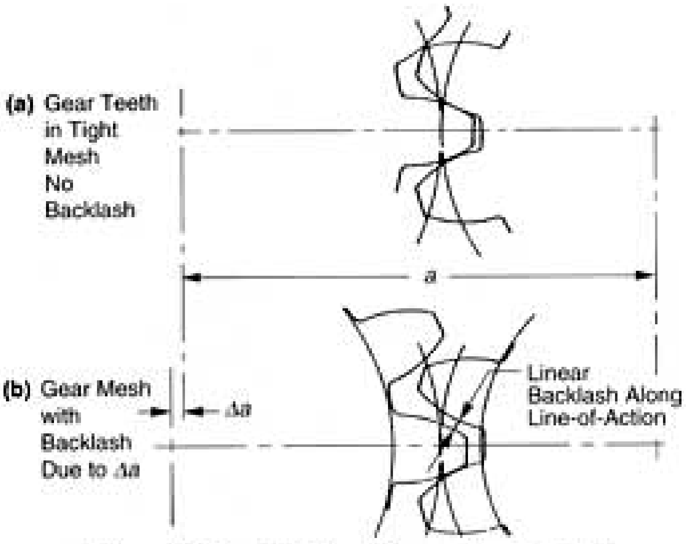


Figure 14-3 Backlash Caused by Opening of Center Distance

This measure along the line-of-action is useful when inserting a feeler gage between teeth to measure backlash. The equivalent linear backlash measured along the pitch circle is given by:

$j = 2\,\Delta a \tan\alpha$ (**14-3a**)

where:

Δa = change in center distance

α = pressure angle

Hence, an approximate relationship between center distance change and change in backlash is:

$\Delta a = 1.933\Delta j$ for 14.5º pressure angle gears (**14-3b**)

$\Delta a = 1.374\Delta j$ for 20º pressure angle gears (**14-3c**)

Although these are approximate relationships, they are adequate for most uses. Their derivation, limitations, and correction factors are detailed in Reference 10.

Note that backlash due to center distance opening is dependent upon the tangent function of the pressure angle. Thus, 20º gears have 41% more backlash than 14.5º gears, and this constitutes one of the few advantages of the lower pressure angle.

Equations (14-3) are a useful relationship, particularly for converting to angular backlash. Also, for fine pitch gears the use of feeler gages for measurement is impractical, whereas an indicator at the pitch line gives a direct measure. The two linear backlashes are related by:

$$j = \frac{j_n}{\cos\alpha} \quad (\textbf{14-4})$$

The angular backlash at the gear shaft is usually the critical factor in the gear application. As seen from **Figure 14-2(a)**, this is related to the gear's pitch radius as follows:

$$j_\theta = 3440\,\frac{j}{R_1} \text{ (arc minutes)} \quad (\textbf{14-5})$$

Obviously, angular backlash is inversely proportional to gear radius. Also, since the two meshing gears are usually of different pitch diameters, the linear backlash of the measure converts to different angular values for each gear. Thus, an angular backlash must be specified with reference to a particular shaft or gear center.

Details of backlash calculations and formulas for various gear types are given in the following sections.

14.2 Backlash Relationships

Expanding upon the previous definition, there are several kinds of backlash: circular backlash J_t, normal backlash j_n, center backlash j_r, and angular backlash J_θ(º), see **Figure 14-4**.

Table 14-1 reveals relationships among circular backlash j_t, normal backlash j_n and center backlash J_r. In this definition, J_r is equivalent to change in center distance, Δa in **Section 14.1**.

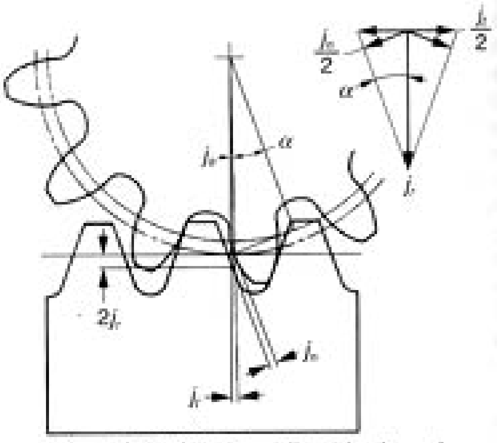


Fig. 14-4 Kinds of Backlash and Their Direction

Table 14-1 The Relationships among the Backlashes

No.	Type of Gear Meshes	The Relation between Circular Backlash j_t and Normal Backlash j_n	The Relation between Circular Backlash j_t and Center Backlash j_r
1	Spur Gear	$j_n = j_t \cos\alpha$	$j_r = \frac{j_t}{2\tan\alpha}$
2	Helical Gear	$j_{nn} = j_{nt} \cos\alpha_n \cos\beta$	$j_r = \frac{j_t}{2\tan\alpha_t}$
3	Straight Bevel Gear	$j_n = j_t \cos\alpha$	$j_r = \frac{j_t}{2\tan\alpha \sin\delta}$
4	Spiral Bevel Gear	$j_{nn} = j_{tt} \cos\alpha_n \cos\beta_m$	$j_r = \frac{j_{tt}}{2\tan\alpha_t \sin\delta}$
5	Worm Worm Gear	$j_{nn} = j_{tt1} \cos\alpha_n \cos\gamma$ $j_{nn} = j_{tt2} \cos\alpha_n \cos\gamma$	$j_r = \frac{j_{tt2}}{2\tan\alpha_x}$

Circular backlash j_t has a relation with angular backlash j_θ as follows:

$$j_\theta = j_t \frac{360}{\pi d} \text{ (degrees)} \qquad \textbf{(14-6)}$$

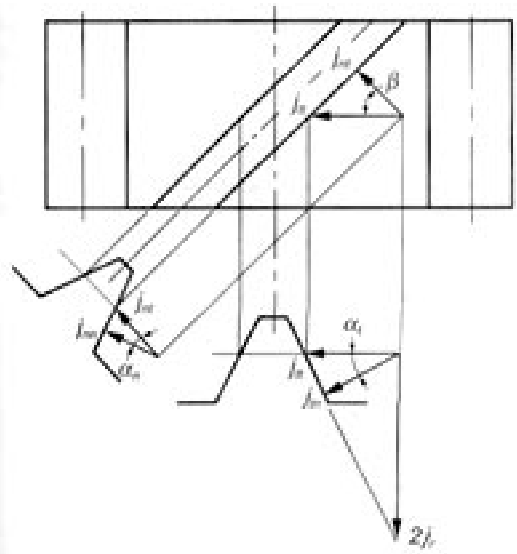
14.2.1 Backlash of a Spur Gear Mesh

From **Figure 14-4** we can derive backlash of sour mesh as:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \qquad \textbf{(14-7)}$$

14.2.2 Backlash of Helical Gear Mesh

The helical gear has two kinds of backlash when referring to the tooth space. There is a cross section in the normal direction of the tooth surface n, and a cross section in the radial direction perpendicular to the axis, t.



j_{nn}= backlash in the direction normal to the tooth surface
j_{nt}= backlash in the circular direction in the cross section normal to the tooth
j_{tn}= backlash in the direction normal to the tooth surface in the cross section perpendicular to the axis
j_{tt} =backlash in the circular direction perpendicular to the axis

Fig. 14-5 Backlash of Helical Gear Mesh

These backlashes have relations as follows:
In the plane normal to the tooth:

$$j_{nn} = j_{nt} \cos\alpha_n \qquad \textbf{(14-8)}$$

On the pitch surface:

$$j_{nt} = j_{tt} \cos\beta \qquad \textbf{(14-9)}$$

In the plane perpendicular to the axis:

$$\left.\begin{aligned} j_{tn} &= j_{tt} \cos\alpha_t \\ j_r &= \frac{j_{tt}}{2\tan\alpha_t} \end{aligned}\right\} \qquad \textbf{(14-10)}$$

14.2.3 Backlash of Straight Bevel Gear Mesh

Figure 14-6 expresses backlash for a straight bevel gear mesh.

In the cross section perpendicular to the tooth of a straight bevel gear, circular backlash at pitch line j_t, normal backlash j_n and radial backlash j_r' have the following relationships:

$$\left.\begin{aligned} j_n &= j_t \cos\alpha \\ j_r' &= \frac{j_t}{2\tan\alpha} \end{aligned}\right\} \qquad \textbf{(14-11)}$$

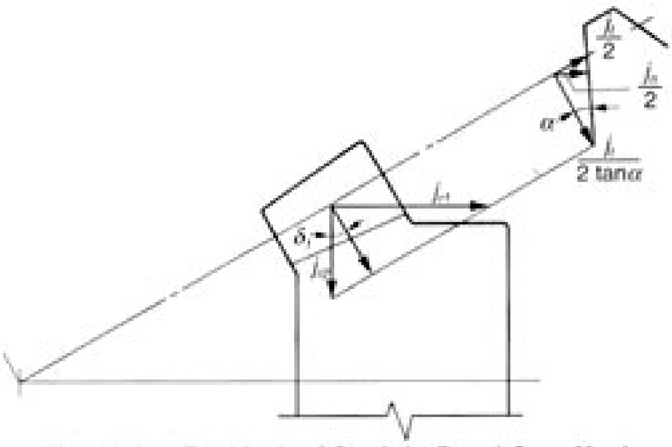


Fig. 14-6 Backlash of Straight Bevel Gear Mesh

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, j_{rt} and in the direction of bevel gear center axis, j_{r2}.

$$\left.\begin{aligned} j_{r1} &= \frac{j_t}{2\tan\alpha \sin\delta_1} \\ j_{r2} &= \frac{j_t}{2\tan\alpha \cos\delta_1} \end{aligned}\right\} \qquad \textbf{(14-12)}$$

14.2.4 Backlash of a Spiral Bevel Gear Mesh

Figure 14-7 delineates backlash for a spiral bevel gear mesh.

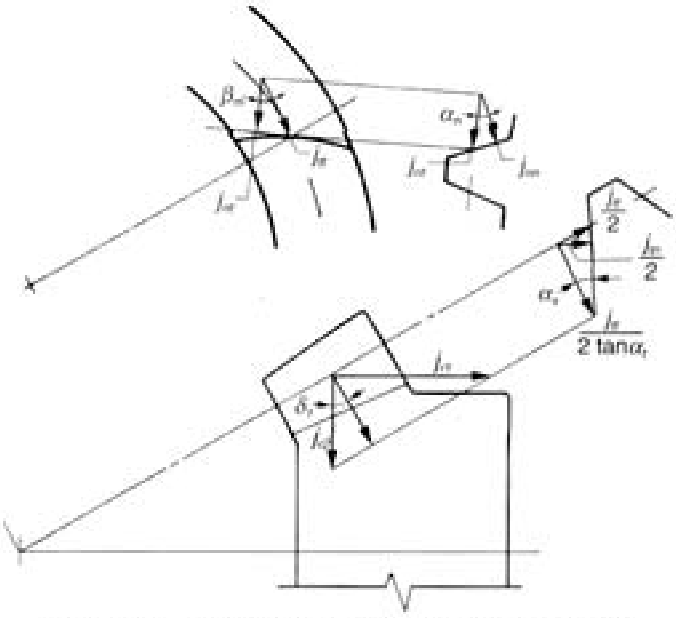


Fig. 14-7 Backlash of Spiral Bevel Gear Mesh

In the tooth space cross section normal to the tooth:

$$j_{nn} = j_{nt} \cos\alpha_n \qquad \textbf{(14-13)}$$

On the pitch surface:

$$j_{nt} = j_{tt} \cos\beta_m \qquad \textbf{(14-14)}$$

In the plane perpendicular to the generatrix of the pitch cone:

$$\left.\begin{aligned} j_{tn} &= j_{tt}\cos\alpha_t \\ j_r' &= \frac{j_{tt}}{2\tan\alpha_t} \end{aligned}\right\} \qquad (\textbf{14-15})$$

The radial backlash in the plane of axes can be broken down into the components in the direction of bevel pinion center axis, and in the direction of bevel gear center axis, j_{r2}

$$\left.\begin{aligned} j_{r1} &= \frac{j_{tt}}{2\tan\alpha_t\sin\delta_1} \\ j_{r2} &= \frac{j_{tt}}{2\tan\alpha_t\cos\delta_1} \end{aligned}\right\} \qquad (\textbf{14-16})$$

14.2.5 Backlash of Worm Gear Mesh

Figure 14-8 expresses backlash for a worm gear mesh.

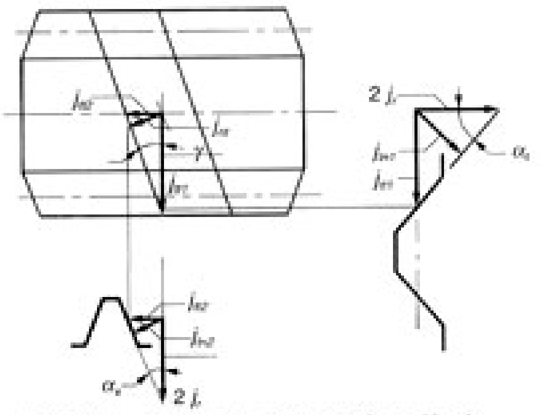


Fig. 14-8 Backlash of Worm Gear Mesh

On the pitch surface of a worm:

$$\left.\begin{aligned} j_{nt} &= j_{tt1}\sin\gamma \\ j_{nt} &= j_{tt2}\cos\gamma \\ \tan\gamma &= \frac{j_{tt2}}{j_{tt1}} \end{aligned}\right\} \qquad (\textbf{14-17})$$

In the cross section of a worm perpendicular to its axis:

$$\left.\begin{aligned} j_{tn1} &= j_{tt1}\cos\alpha_t \\ j_r &= \frac{j_{tt1}}{2\tan\alpha_t} \end{aligned}\right\} \qquad (\textbf{14-18})$$

In the plane pependicular to the axis of the worm gear:

$$\left.\begin{aligned} j_{tn2} &= j_{tt2}\cos\alpha_x \\ j_r &= \frac{j_{tt2}}{2\tan\alpha_x} \end{aligned}\right\} \qquad (\textbf{14-19})$$

14.3 Tooth Thickness And Backlash

There are two ways to produce backlash. One is to enlarge the center distance. The other is to reduce the tooth thickness. The latter is much more popular than the former. We are going to discuss more about the way of reducing the tooth thickness. In **SECTION 10**, we have discussed the standard tooth thickness s. In the meshing of a pair of gears, if the tooth thickness of pinion and gear were reduced by ΔS_1 and ΔS_2 they would produce a backlash of $\Delta s_1 + \Delta s_2$ in the direction of the pitch circle.

Let the magnitude of Δs_1 Δs_2 be 0.1. We know that α = 20º then:

$j_t = \Delta S_1 + \Delta S_2 = 0.1 + 0.1 = 0.2$

We can convert it into the backlash on normal direction:

$j_n = j_t\cos\alpha = 0.2\cos 20º = 0.1879$

Let the backlash on the center distance direction be j_n then:

$$j_r = \frac{j_t}{2\tan\alpha} = \frac{0.2}{2\tan 20º} = 0.2747$$

These express the relationship among several kinds of backlashes. In application, one should consult the JIS standard.

There are two JIS standards for backlash - one is JIS B 1703-76 for spur gears and helical gears, and the other is JIS B 1705-73 for bevel gears. All these standards regulate the standard backlashes in the direction of the pitch circle j_t or j_{tt}. These standards can be applied directly, but the backlash beyond the standards may also be used for special purposes. When writing tooth thicknesses on a drawing, it is necessary to specify, in addition, the tolerances on the thicknesses as well as the backlash. For example:

Circular tooth thickness	$3.141^{-0.050}_{-0.100}$
Backlash	0.100 ... 0.200

14.4 Gear Train And Backlash

The discussions so far involved a single pair of gears. Now, we are going to discuss two stage gear trains and their backlash. In a two stage gear train, as **Figure 14-9** shows, j_1 and j_4 represent the backlashes of first stage gear train and second stage gear train respectively.

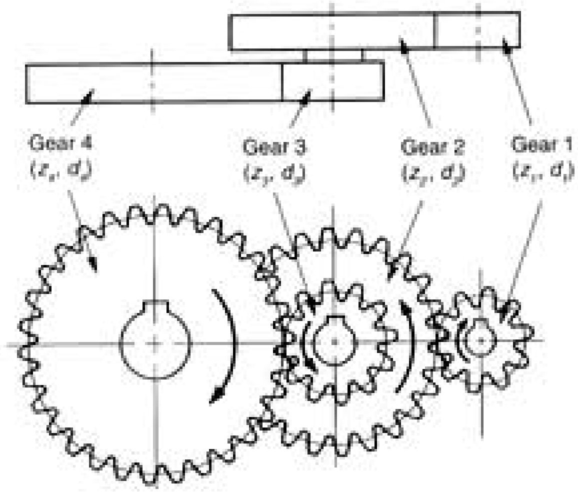


Fig. 14-9 Overall Accumulated Backlash of Two Stage Gear Train

If number one gear were fixed, then the accumulated backlash on number four gear j_{tT4} would be as follows:

$$j_{tT4} = j_1\frac{d_3}{d_2} + j_4 \qquad (\textbf{14-20})$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT4}\frac{360}{\pi d_4}\text{ (degrees)} \qquad (\textbf{14-21})$$

The reverse case is to fix number four gear and to examine the accumulated backlash on number one gear j_{tT1}

$$j_{tT1} = j_4\frac{d_2}{d_3} + j_1 \qquad (\textbf{14-22})$$

This accumulated backlash can be converted into rotation in degrees:

$$j_\theta = j_{tT1}\frac{360}{\pi d_1}\text{ (degrees)} \qquad (\textbf{14-23})$$

14.5 Methods Of Controlling Backlash

In order to meet special needs, precision gears are used more Frequently than ever before. Reducing backlash becomes an important issue. There are two methods of reducing or eliminating backlash - one a static, and the other a dynamic method.

The static method concerns means of assembling gears and then making proper adjustments to achieve the desired low backlash. The dynamic method introduces an external force which continually eliminates all backlash regardless of rotational position.

14.5.1 Static Method

This involves adjustment of either the gears effective tooth thickness or the mesh center distance. These two independent adjustments can be used to produce four possible combinations as shown in **Table 14-2**.

Table 14-2

		Center Distance	
		Fixed	Adjustable
Gear Size	Fixed	I	III
	Adjustable	II	IV

Case I

By design, center distance and tooth thickness are such that they yield the proper amount of desired minimum backlash. Center distance and tooth thickness size are fixed at correct values and require precision manufacturing.

Case II

With gears mounted on fixed centers, adjustment is made to the effective tooth thickness by axial movement or other means. Three main methods are:

1. Two identical gears are mounted so that one can be rotated relative to the other and fixed. See **Figure 14-10a**. In this way, the effective tooth thickness can be adjusted to yield the desired low backlash.
2. A gear with a helix angle such as a helical gear is made in two half thicknesses. One is shifted axially such that each makes contact with the mating gear on the opposite sides of the tooth. See **Figure 14-10b**.
3. The backlash of cone shaped gears, such as bevel and tapered tooth spur gears, can be adjusted with axial positioning. A duplex lead worm can be adjusted similarly. See **Figure 14-10c**.

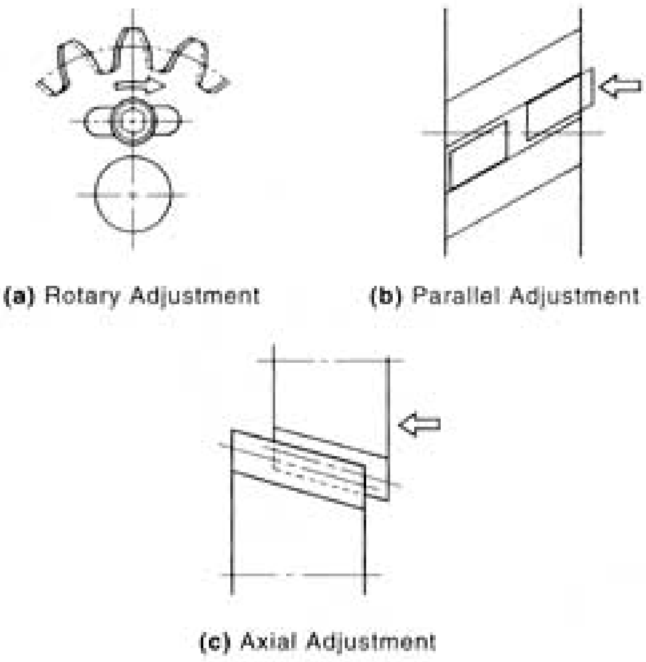


Fig. 14-10 Ways of Decreasing Backlash in Case II

Case III

Center distance adjustment of backlash can be accomplished in two ways:

1. Linear Movement - **Figure 14-11a** show adjustment along the line-of-centers in a straight or parallel axes manner. After setting to the desire value of backlash the centers are locked in place.
2. Rotary Movement- **Figure 14-11b** show an alternate way c achieving center distance adjustment b rotation of one of the gear centers b means of a swing arm on an eccentric bushing. Again, once the desired backlash setting is found, the positioning arm is locked.

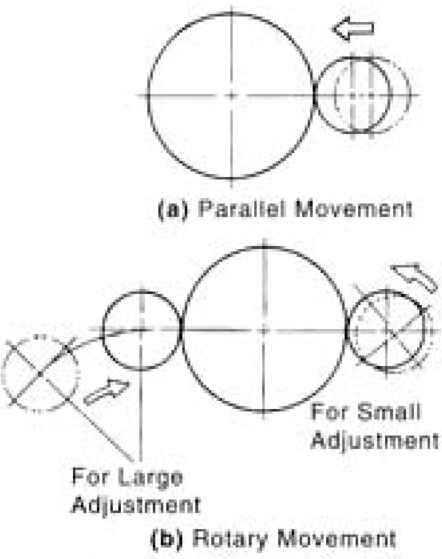


Fig. 14-11 Ways of Decreasing Backlash in Case III

Case IV

Adjustment of both center distance and tooth thickness is theoretically valid, but is not the usual practice. This would call for needless fabrication expense.

14.5.2 Dynamic Methods

Dynamic methods relate to the static techniques. However, they involve a forced adjustment of either the effective tooth thickness or the center distance.

1. Backlash Removal by Forced Tooth Contact

This is derived from static Case 11 Referring to **Figure 14-10a**. a forcing spring rotates the two gear halves apart. This results in an effective tooth thickness that continually fills the entire tooth space in all mesh positions.

2. Backlash Removal by Forced Center Distance Closing

This is derived from static Case lll. A spring force is applied to close the center distance; in one case as a linear force along the line-of-centers, and in the other case as a torque applied to the swing arm.

In all of these dynamic methods, the applied external force should be known and properly specified. The theoretical relationship of the forces involved is as follows:

$$F > F_1 + F_2 \quad \textbf{(14-24)}$$

where:

F_1 = Transmission Load on Tooth Surface

F_2 = Friction Force on Tooth Surface

If $F < F_1 + F_2$, then it would be impossible to remove backlash. But if F is excessively greater than a proper level, the tooth surfaces would be needlessly loaded and could lead to premature wear and shortened life. Thus, in designing such gears, consideration must be given to not only the needed transmission load, but also the forces acting upon the tooth surfaces caused by the spring load. It is important to appreciate that the spring loading must be set to accommodate the largest expected transmission force, F_1. and this maximum spring force is applied to the tooth surfaces continually and irrespective of the load being driven.

3. Duplex Lead Worm

A duplex lead worm mesh is a special design in which backlash can be adjusted by shifting the worm axially. it is useful for worm