12.2 Crowning And Side Relieving

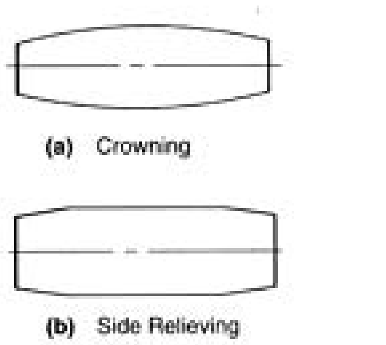

Fig. 12-2 Crowning and Relieving

Crowning and side relieving are tooth surface modifications in the axial direction. See **Figure 12-2**.

Crowning is the removal of a slight amount of tooth from the center on out to reach edge, making the tooth surface slightly convex. This method allows the gear to maintain contact in the central region of the tooth and permits avoidance of edge contact with consequent lower load capacity. Crowning also allows a greater tolerance in the misalignment of gears in their assembly, maintaining central contact.

Relieving is a chamfering of the tooth surface. It is similar to crowning except that it is a simpler process and only an approximation to crowning. It is not as effective as crowning.

12.3 Topping And Semitopping

In topping, often referred to as top hobbing, the top or outside diameter of the gear is cut simultaneously with the generation of the teeth. An advantage is that there will be no burrs on the tooth top. Also, the outside diameter is highly concentric with the pitch circle. This permits secondary machining operations using this diameter for nesting.

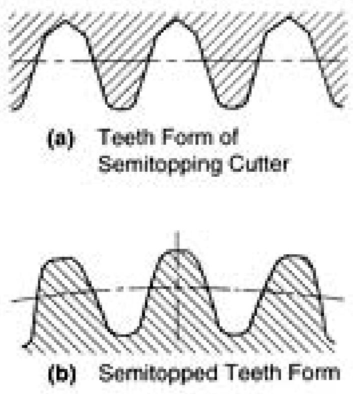

Fig. 12-3 Semitopping Cutter and the Gear Profile Generated

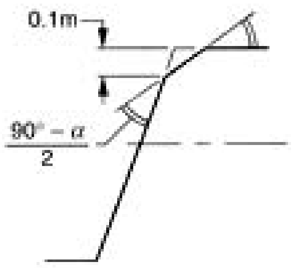

Fig. 12-4 Recommended Magnitude of Semitopping

Semitopping is the chamfering of the tooth's top corner, which is accomplished simultaneously with tooth generation. **Figure 12-3** shows a semitopping cutter and the resultant generated semitopped gear. Such a tooth tends to prevent corner damage. Also, it has no burr. The magnitude of semitopping should not go beyond a proper limit as otherwise it would significantly shorten the addendum and contact ratio. **Figure 12-4** specifies a recommended magnitude of semitopping.

Both modifications require special generating tools. They are independent modifications but, if desired, can be applied simultaneously.

SECTION 13 GEAR TRAINS

The objective of gears is to provide a desired motion, either rotation or linear. This is accomplished through either a simple gear pair or a more involved and complex system of several gear meshes. Also, related to this is the desired speed, direction of rotation and the shaft arrangement.

13.1 Single-Stage Gear Train

A meshed gear is the basic form of a single-stage gear train. It consists of z_1 and z_2 numbers of teeth on the driver and driven gears, and their respective rotations, n_1 & n_2 The speed ratio is then:

$$\text{speed ratio} = \frac{z_1}{z_2} = \frac{n_2}{n_1} \qquad (\mathbf{13\text{-}1})$$

13.1.1 Types of Single-Stage Gear Trains

Gear trains can be classified into three types:

1. Speed ratio > 1, increasing: $n_1 < n_2$
2. Speed ratio =1, equal speeds: $n_1 = n_2$
3. Speed ratio < 1, reducing: $n_1 > n_2$

Figure 13-1 illustrates four basic types. For the very common cases of spur and bevel meshes, **Figures 13-1(a)** and **13-1(b)**, the direction of rotation of driver and driven gears are reversed. In the case of an internal gear mesh, **Figure 13-1(c)**, both gears have the same direction of rotation. In the case of a worm mesh, **Figure 13-1(d)**, the rotation direction of z_2 is determined by its helix hand.

In addition to these four basic forms, the combination of a rack and gear can be considered a specific type. The displacement of a rack, ι, for rotation 0 of the mating gear is:

$$\iota = \frac{\pi m z_1 \phi}{360} \qquad (\mathbf{13\text{-}2})$$

where:
π m is the standard circular pitch
z_1 is the number of teeth of the gear

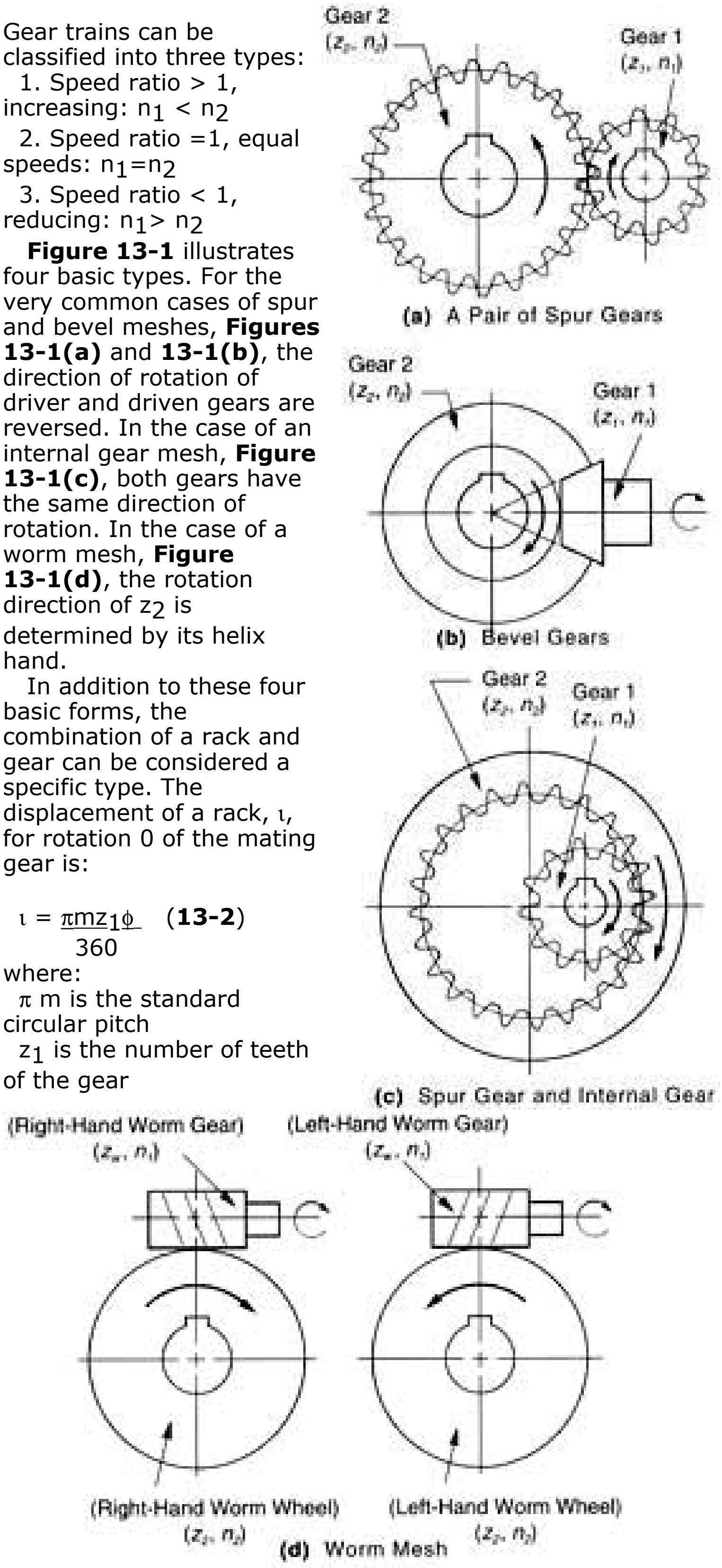

Fig. 13-1 Single-Stage Gear Trains

13.2 Two-Stage Gear Train

A two-stage gear train uses two single-stages in a series. **Figure 13-2** represents the basic form of an external gear two-stage gear train.

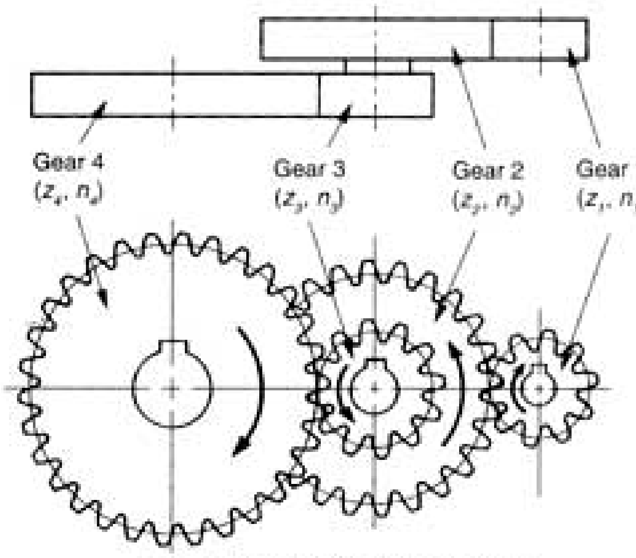

Fig. 13-2 Two-Stage Gear Train

Let the first gear in the first stage be the driver. Then the speed ratio of the two-stage train is:

Speed Ratio = $\frac{z_1}{z_2}\frac{z_3}{z_4}\frac{n_2}{n_1}\frac{n_4}{n_3}$ (**13-3**)

In this arrangement, $n_2 = n_3$

In the two-stage gear train, **Figure 13-2**. gear 1 rotates in the same direction as gear 4. If gears 2 and 3 have the same number of teeth, then the train simplifies as in **Figure 13-3**. In this arrangement, gear 2 is known as an idler, which has no effect on the gear ratio.
The speed ratio is then:

Speed Ratio = $\frac{Z_1}{Z_2}\frac{Z_2}{Z_3} = \frac{Z_1}{Z_3}$ (**13-4**)

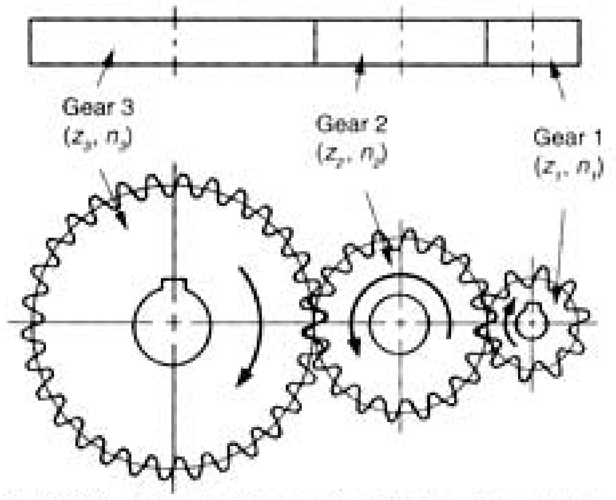

Fig. 13-3 Single-Stage Gear Train with an Idler

13.3 Planetary Gear System

The basic form of a planetary gear system is shown in **Figure 13-4**. It consists of a sun gear A, planet gears B, internal gear C and carrier D. The input and output axes of a planetary gear system are on a same line. Usually, it uses two or more planet gears to balance the load evenly. It is compact in space, but complex in structure. Planetary gear systems need a high-quality manufacturing process. The load division between planet gears, the interference of the internal gear, the balance and vibration of the rotating carrier, and the hazard of jamming, etc. are inherent problems to be solved.

Figure 13-4 is a so called 2K-H type planetary gear system The sun gear, internal gear. and the carrier have a common axis.

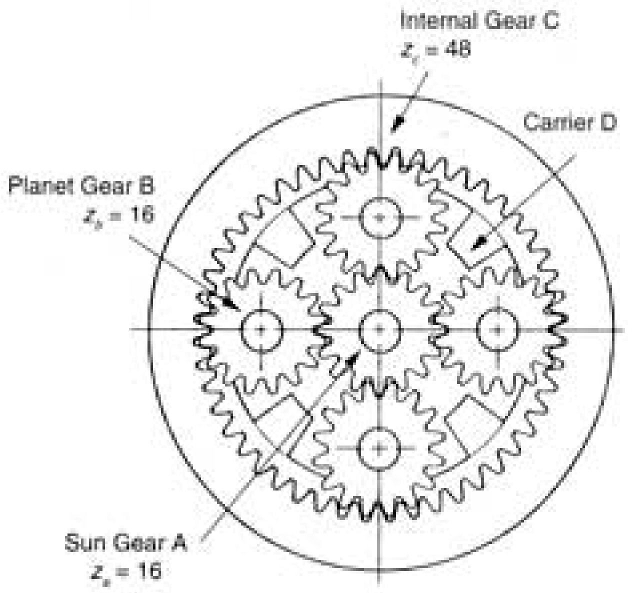

Fig. 13-4 An Example of a Planetary Gear System

13.3.1 Relationship Among the Gears in a Planetary Gear System

In order to determine the relationship among the numbers of teeth of the sun gear A, z_a, the planet gears B, z_b and the internal gear C, z_c and the number of planet gears, N, in the system, the parameters must satisfy the following three conditions:

Condition No. 1:

$Z_c = Z_a + 2 Z_b$ (**13-5**)

This is the condition necessary for the center distances of the gears to match. Since the equation is true only for the standard gear system, it is possible to vary the numbers of teeth by using profile shifted gear designs.

To use profile shifted gears, it is necessary to match the center distance between the sun A and planet B gears, a_{x1}, and the center distance between the planet B and internal C gears, a_{x2}

$a_{x1} = a_{x2}$ (**13-6**)

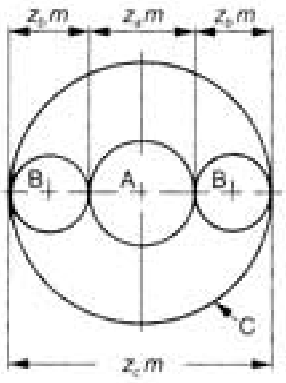

Fig. 13-5(a) Condition No. 1 of Planetary Gear System

Condition No. 2:

$\frac{(z_a + z_c)}{N}$=integer (**13-7**)

This is the condition necessary For placing planet gears evenly spaced around the sun gear. If an uneven placement of planet gears is desired, then **Equation (13-8)** must be satisfied.

$\frac{(z_a+z_c)\theta}{180}$=Integer (**13-8**)

where:

θ = half the angle between adjacent planet gears

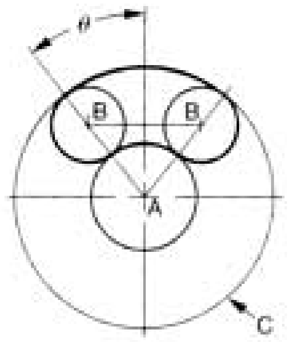

Fig. 13-5(b) Condition No. 2 of Planetary Gear System

Condition No. 3:

$$\text{Speed Ratio} = \frac{\frac{z_a}{z_c}}{1+\frac{z_a}{z_c}} = \frac{1}{\frac{z_c}{z_a}+1} \qquad (13\text{-}9)$$

Satisfying this condition insures that adjacent planet gears can operate without interfering with each other. This is the condition that must be met for standard gear design with equal placement of planet gears. For other conditions, the system must satisfy the relationship:

$d_{ab} < 2\, a_x \sin\theta$ (**13-10**)

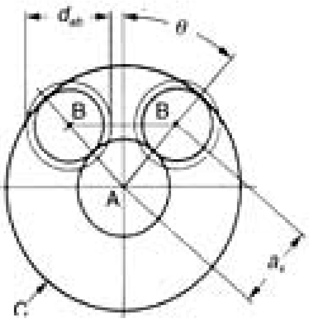

Fig. 13-5(c) **Condition No. 3 of Planetary Gear System**

where:

d_{ab} = outside diameter of the planet gears

a_x = center distance between the sun and planet gears

Besides the above three basic conditions, there can be an interference problem between the internal gear C and the planet gears B. See **SECTION 5** that discusses more about this problem.

13.3.2 Speed Ratio of Planetary Gear System

In a planetary gear system, the speed ratio and the direction of rotation would be changed according to which member is fixed. **Figures 13-6(a), 13-6(b)** and **136(c)** contain three typical types of planetary gear mechanisms, depending upon which member is locked.

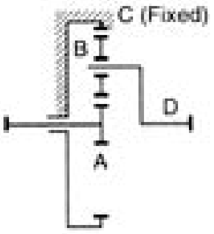

Fig. 13-6(a) **Planetary Type Planetary Gear Mechanism**

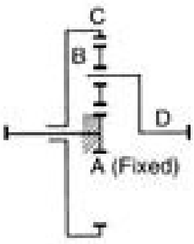

Fig. 13-6(b) **Solar Type Planetary Gear Mechanism**

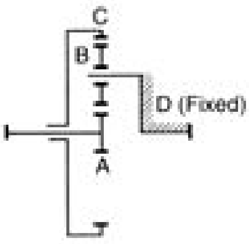

Fig. 13-6(c) **Star Type Planetary Gear Mechanism**

(a) Planetary Type

In this type, the internal gear is fixed. The input is the sun gear and the output is carrier D. The speed ratio is calculated as in **Table 13-1**.

$$\text{Speed Ratio} = \frac{\frac{z_a}{z_c}}{1+\frac{z_a}{z_c}} = \frac{1}{\frac{z_c}{z_a}+1} \qquad (\mathbf{13\text{-}11})$$

Note that the direction of rotation of input and output axes are the same.

Example: Z_a=16, Z_b=16, Z_c=48, then speed ratio=1/4.

(b) Solar Type

In this type, the sun gear is fixed. The internal gear C is the input, and carrier D axis is the output. The speed ratio is calculated as in **Table 13-2**.

$$\text{Speed Ratio} = \frac{-1}{-\frac{z_a}{z_c}-1} = \frac{1}{\frac{z_a}{z_c}+1} \qquad (\mathbf{13\text{-}12})$$

Note that the directions of rotation of input and output axes are the same.

Example: Z_a = 16, z_b = 16, z_c = 48, then the speed ratio = 1/1.3333333.

(**c**) Star Type

This is the type in which Carrier D is fixed. The planet gears rotate only on fixed axes. In a strict definition, this train, loses the features of a planetary system and it becomes an ordinary gear train. The sun gear is an input axis and the internal gear is the output. The speed ratio is:

$$\text{Speed Ratio} = -\frac{z_a}{z_c} \qquad (\mathbf{13\text{-}13})$$

Referring to **Figure 13-6(c)**, the planet gears are merely idlers. Input and output axes have opposite rotations.

Example: Z_a = 16, Z_b = 16, Z_c = 48;

then speed ratio = - 1/3.

13.4 Constrained Gear System

A planetary gear system which has four gears, as in **Figure 13-5**, is an example of a constrained gear system. It is a closed loop system in which the power is transmitted from the driving gear through other gears and eventually to the driven gear. A closed loop gear system will not work if the gears do not meet specific conditions.

Let z_1, z_2 and z_3 be the numbers of gear teeth, as in **Figure 13-7**. Meshing cannot function if the length of the heavy line (belt) does not divide evenly by circular pitch. **Equation (13-14)** defines this condition.

$$\frac{z_1\theta_1}{180}+\frac{Z_2(180+z\theta_1+\theta_2)}{180}+\frac{z_3\theta_2}{180}=\text{Integer} \qquad (\mathbf{13\text{-}14})$$

where θ_1 and θ_2 are in degrees.

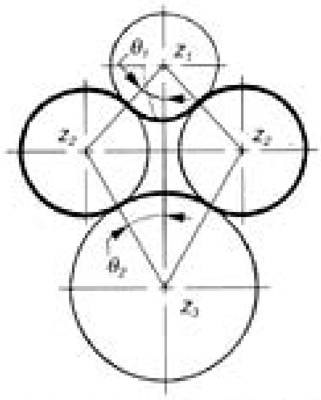

Fig. 13-7 **Constrained Gear System**

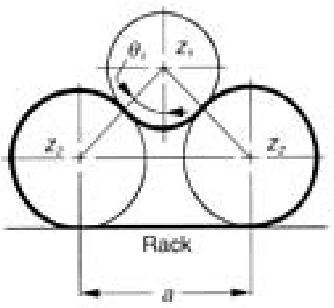

Fig. 13-8 **Constrained Gear System Containing a Rack**

Figure 13-8 shows a constrained gear system in which a rack is meshed. The heavy line in **Figure 13-8** corresponds to the belt in **Figure 13-7**. If the length of the belt cannot be evenly divided by circular pitch then the system does not work. It is described by **Equation (13-15)**.

$$\frac{z_1\theta_1}{180} + \frac{z_2(180+\theta_1)}{180} + \frac{a}{\pi m} = \text{integer} \qquad (\mathbf{13\text{-}15})$$