4.7 Rack And Spur Gear

Table 4-6 presents the method for calculating the mesh of a rack arid spur gear. **Figure 4-9a** shows the pitch circle of a standard gear and the pitch line of the rack.

One rotation of the spur gear will displace the rack one circumferential length of the gear's pitch circle, per the formula:

$$\iota = \pi m z \qquad \textbf{(4-6)}$$

Figure 4-9b shows a profile shifted spur gear, with positive correction xm, meshed with a rack. The spur gear has a larger pitch radius than standard, by the amount xm. Also, the pitch line of the rack has shifted outward by the amount xm.

Table 4-6 presents the calculation of a meshed profile shifted spur gear and rack. If the correction factor x_1, is 0, then it is the case of a standard gear meshed with the rack.

The rack displacement, ι, is not changed in any way by the profile shifting. **Equation (4-6)** remains applicable for any amount of profile shift.

Table 4-6 The Calculation of Dimensions of a Profile Shifted Spur Gear and a Rack

No.	Item	Symbol	Formula	Example: Spur Gear	Example: Rack
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z		12	—
4	Coefficient of Profile Shift	x		0.6	
5	Height of Pitch Line	H		—	32.000
6	Working Pressure Angle	α_w		20°	
7	Center Distance	a_x	$\frac{zm}{2} + H + xm$	51.800	
8	Pitch Diameter	d	zm	36.000	—
9	Base Diameter	d_b	$d \cos\alpha$	33.829	
10	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	36.000	
11	Addendum	h_a	$m(1 + x)$	4.800	3.000
12	Whole Depth	h	$2.25m$	6.750	
13	Outside Diameter	d_a	$d + 2h_a$	45.600	—
14	Root Diameter	d_f	$d_a - 2h$	32.100	

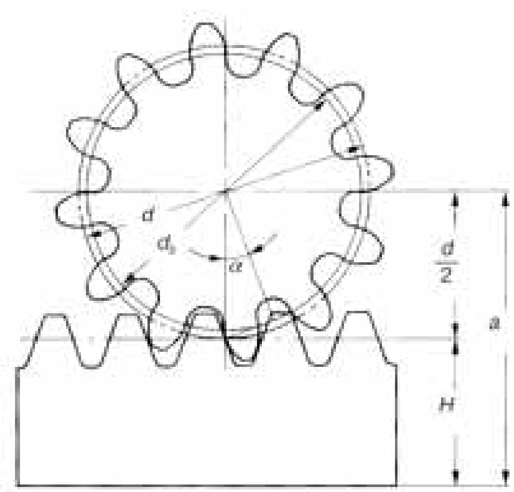

Fig. 4-9a The Meshing of Standard Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = 0$)

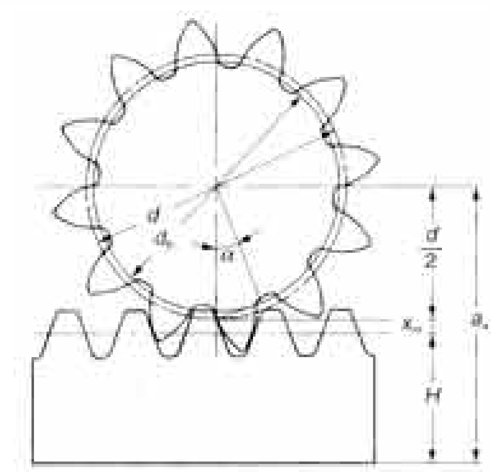

Fig. 4-9b The Meshing of Profile Shifted Spur Gear and Rack
($\alpha = 20°$, $z_1 = 12$, $x_1 = +0.6$)

SECTION 5 INTERNAL GEARS

5.1 Internal Gear Calculations
Calculation of a Profile Shifted Internal Gear

Figure 5-1 presents the mesh of an internal gear and external gear. Of vital importance is the operating (working) pitch diameters, d_w, and operating (working) pressure angle, α_w They can be derived from center distance, a_x, and **Equations (5-1)**.

$$\left.\begin{aligned} d_{w1} &= 2a_x \frac{z_1}{z_2 - z_1} \\ d_{w2} &= 2a_x \frac{z_2}{z_2 - z_1} \\ \alpha_w &= \cos^{-1}\left(\frac{d_{b2} - d_{b1}}{2a_x}\right) \end{aligned}\right\} \qquad (5\text{-}1)$$

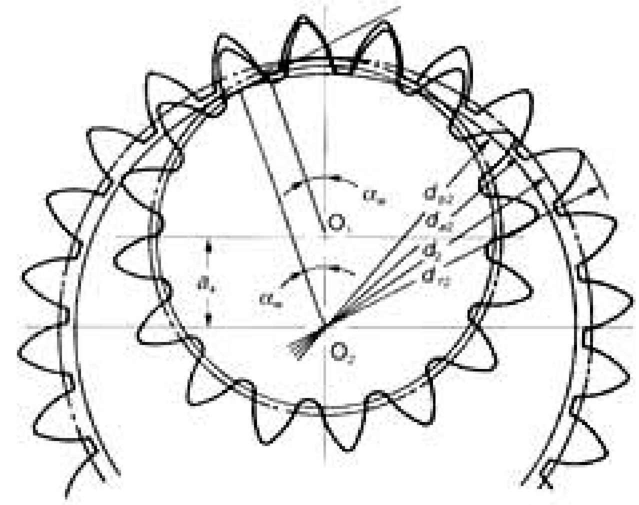

Fig. 5-1 The Meshing of Internal Gear and External Gear
($\alpha = 20^\circ$, $z_1 = 16$, $z_2 = 24$, $x_1 = x_2 = 0.5$)

Table 5-1 shows the calculation steps. It will become a standard gear calculation if

$x_1 = x_2 = 0$.

If the center distance, a_x is given, x_1 and x_2 would be obtained from the inverse calculation from item 4 to item 8 of **Table 5-1**. These inverse formulas are in **Table 5-2**.

Pinion cutters are often used in cutting internal gears and external gears. The actual value of tooth depth and root diameter, after cutting, will be slightly different from the calculation. That is because the cutter has a coefficient of shifted profile. In order to get a correct tooth profile, the coefficient of cutter should be taken into consideration.

5.2 Interference In Internal Gears

Three different types of interference can occur with internal gears:

(a) Involute Interference
(b) Trochoid Interference
(c) Trimming Interference

(a) Involute Interference

This occurs between the dedendum of the external gear and the addendum of the internal gear. It is prevalent when the number of teeth of the external gear is small. Involute interference can be avoided by the conditions cited below:

$$\frac{z_1}{z_2} \geq 1 - \frac{\tan\alpha_{a2}}{\tan\alpha_w} \qquad (5\text{-}2)$$

where α_{a2} is the pressure angle seen at a lip of the internal gear tooth.

$$\alpha_{a2} = \cos^{-1}\left(\frac{d_{b2}}{d_{a2}}\right) \qquad (5\text{-}3)$$

and α_w is working pressure angle:

$$\alpha_w = \cos^{-1}\left[\frac{(z_2 - z_1)\, m\cos\alpha}{2a_x}\right] \qquad (5\text{-}4)$$

Equation (5-3) is true only if the Outside diameter of the internal gear is bigger than the base circle:

$$d_{a2} \geq d_{b2} \qquad (5\text{-}5)$$

Table 5-1 The Calculation of a Profile Shifted Internal Gear and External Gear (1)

No.	Item	Symbol	Formula	Example: External Gear	Example: Internal Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		16	24
4	Coefficient of Profile Shift	x_1, x_2		0	0.5
5	Involute Function α	$\mathrm{inv}\alpha$	$2\tan\alpha\left(\frac{x_2 - x_1}{z_2 - z_1}\right) + \mathrm{inv}\alpha$	0.060401	
6	Working Pressure Angle	α_w	Find from Involute Function Table	31.0937°	
7	Center Distance Increment Factor	y	$\frac{z_2 - z_1}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.389426	
8	Center Distance	a_x	$\left(\frac{z_2 - z_1}{2} + y\right)m$	13.1683	
9	Pitch Diameter	d	zm	48.000	72.000
10	Base Circle Diameter	d_b	$d\cos\alpha$	45.105	67.658
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	52.673	79.010
12	Addendum	h_{a1} h_{a2}	$(1 + x_1)m$ $(1 - x_2)m$	3.000	1.500
13	Whole Depth	h	$2.25m$	6.75	
14	Outside Diameter	d_{a1} d_{a2}	$d_1 + 2h_{a1}$ $d_2 - 2h_{a2}$	54.000	69.000
15	Root Diameter	d_{f1} d_{f2}	$d_{a1} - 2h$ $d_{a2} + 2h$	40.500	82.500

Table 5-2 The Calculation of Shifted Internal Gear and External Gear (2)

No.	Item	Symbol	Formula	Example	
1	Center Distance	a_x		13.1683	
2	Center Distance Increment Factor	y	$\frac{a_x}{m} - \frac{z_2 - z_1}{2}$	0.38943	
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_2 - z_1)\cos\alpha}{2y + z_2 - z_1}\right]$	31.0937°	
4	Difference of Coefficients of Profile Shift	$x_2 - x_1$	$\frac{(z_2 - z_1)(\mathrm{inv}\alpha_w - \mathrm{inv}\alpha)}{2\tan\alpha}$	0.5	
5	Coefficient of Profile Shift	x_1, x_2		0	0.5

For a standard internal gear, where $\alpha = 20^o$, **Equation (5-5)** is valid only if the number of teeth is $z_2 > 34$.

(b) Trochoid Interference

This refers to an interference occurring at the addendum of the external gear and the dedendum of the internal gear during recess tooth action, It tends to happen when the difference between the numbers of teeth of the two gears is small. **Equation (5-6)** presents the condition for avoiding trochoidal Interference.

$$\theta_1 \frac{z_1}{z_2} + \text{inv}\alpha_w - \text{inv}\alpha_{a2} \geq \theta_2 \qquad (5\text{-}6)$$

Here

$$\left.\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{r_{a2}^2 - r_{a1}^2 - a^2}{2ar_{a1}}\right) + \text{inv}\,\alpha_{a1} - \text{inv}\,\alpha_w \\ \theta_2 &= \cos^{-1}\left(\frac{a^2 + r_{a2}^2 - r_{a1}^2}{2ar_{a2}}\right) \end{aligned}\right\} \qquad (5\text{-}7)$$

where α_{a1} is the pressure angle of the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad (5\text{-}8)$$

where α_{a1} is the pressure angle or the spur gear tooth tip:

$$\alpha_{a1} = \cos^{-1}\left(\frac{d_{b1}}{d_{a1}}\right) \qquad (5\text{-}8)$$

In the meshing of an external gear and a standard internal gear $\alpha = 20^o$, trochoid interference is avoided if the difference of the number of teeth, $z_1 - z_2$, is larger than 9.

(c) Trimming Interference

This occurs in the radial direction in that it prevents pulling the gears apart. Thus, the mesh must be assembled by sliding the gears together with an axial motion. It tends to happen when the numbers of teeth of the two gears are very close. **Equation (5-9)** indicates how to prevent this type of interference.

$$\theta_1 + \text{inv}\alpha_{a1} - \text{inv}\alpha_w \geq \frac{z_2}{z_1}\left(\theta_2 + \text{inv}\alpha_{a2} - \text{inv}\alpha_w\right) \qquad (5\text{-}9)$$

Here

$$\left.\begin{aligned} \theta_1 &= \sin^{-1}\sqrt{\frac{1 - (\cos\alpha_{a1}/\cos\alpha_{a2})^2}{1 - (z_1/z_2)^2}} \\ \theta_2 &= \sin^{-1}\sqrt{\frac{(\cos\alpha_{a2}/\cos\alpha_{a1})^2 - 1}{(z_2/z_1)^2 - 1}} \end{aligned}\right\} \qquad (5\text{-}10)$$

This type of interference can occur in the process of cutting an internal gear with a pinion cutter. Should that happen, there is danger of breaking the tooling. **Table 5-3a** shows the limit for the pinion cutter to prevent trimming interference when cutting a standard internal gear, with pressure angle 20º, and no profile shift, i.e., $x_c = 0$.

Table 5-3a The Limit to Prevent an Internal Gear from Trimming Interference ($\alpha = 20^o$, $x_c = x_2 = 0$)

zc	15	16	17	18	19	20	21	22	24	25	27
z2	34	34	35	36	37	38	39	40	42	43	45
zc	28	30	31	32	33	34	35	38	40	42	
z2	46	48	49	50	51	52	53	56	58	60	
zc	44	48	50	56	60	64	66	80	96	100	
z2	62	66	68	74	78	82	84	98	114	118	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 22 (z_c = 15 to 22). **Table 5-3b** shows the limit for a profile shifted pinion cutter to prevent trimming interference while cutting a standard internal gear. The correction, x_c is the magnitude of shift which was assumed to be: $x_c = 0.0075\, z_c + 0.05$.

Table 5-3b The Limit to Prevent an Internal Gear from Trimmina Interference (a = 20ø. x2 = 0)

z_c	15	16	17	18	19	20	21	22	24	25	27
x_c	0.1625	0.17	0.1775	0.185	0.1925	0.2	0.2075	0.215	0.23	0.2375	0.2525
z_2	36	38	39	40	41	42	43	45	47	48	50
z_c	28	30	31	32	33	34	35	38	40	42	
x_c	0.26	0.275	0.2825	0.29	0.2975	0.305	0.3125	0.335	0.35	0.365	
z_2	52	54	55	56	58	59	60	64	66	68	
z_c	44	48	50	56	60	64	66	80	96	100	
x_c	0.38	0.41	0.425	0.47	0.5	0.53	0.545	0.65	0.77	0.8	
z_2	71	76	78	86	90	95	98	115	136	141	

There will be an involute interference between the internal gear and the pinion cutter if the number of teeth of the pinion cutter ranges from 15 to 19 (z_c= 15 to 19).

5.3 Internal Gear With Small Differences In Numbers Of Teeth

In the meshing of an internal gear and an external gear, if the difference in numbers of teeth of two gears is quite small, a profile shifted gear could prevent the interference. **Table 5-4** is an example of how to prevent interference under the conditions of $z_2 = 50$ and the difference of numbers of teeth of two gears ranges from 1 to 8.

Table 5-4 The Meshing of Internal and External Gears of Small Difference of Numbers of Teeth (m=1, a=20º)

z_1	49	48	47	46	45	44	43	42
x_1	0							
z_2	50							
x_2	1.00	0.60	0.40	0.30	0.20	0.11	0.06	0.01
α_w	61.0605º	46.0324º	37.4155º	32.4521º	28.4521º	24.5356º	22.3755º	20.3854º
a	0.971	1.354	1.775	2.227	2.666	3.099	3.557	4.010
ε	1.105	1.512	1.726	1.835	1.933	2.014	2.053	2.088

All combinations above will not cause involute interference or trochoid interference, but trimming interference is still there. In order to assemble successfully, the external gear should be assembled by inserting in the axial direction.

A profile shifted internal gear and external gear, in which the difference of numbers of teeth is small, belong to the field of hypocyclic mechanism, which can produce a large reduction ratio in one step, such as 1/100.

$$\text{Speed Ratio} = \frac{Z_2 - Z_1}{z_1} \qquad (5\text{-}11)$$