3.21 Contact Ratio

To assure smooth continuous tooth action, as one pair of teeth ceases contact a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as possible. The measure of this overlapping is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch. Figure 3-3 shows the geometry. The length-of-action is determined from the intersection of the line-of-action and the outside radii. For the simple case of a pair of spur gears, the ratio of the length of-action to the base pitch is determined from:

$$\varepsilon_\gamma = \frac{\sqrt{(R_a^2 - R_b^2)} + \sqrt{(r_a^2 - r_b^2)} - a\sin\alpha}{p\cos\alpha} \qquad \textbf{(3-4)}$$

It is good practice to maintain a contact ratio of 1 .2 or greater. Under no circumstances should the ratio drop below 1.1, calculated for all tolerances at their worst-case values.

A contact ratio between 1 and 2 means that part of the time two pairs of teeth are in contact and during the remaining time one pair is in contact. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact. Such a high contact ratio generally is not obtained with external spur gears, but can be developed in the meshing of an internal and external spur gear pair or specially designed nonstandard external spur gears.

More detail is presented about contact ratio, including calculation equations for specific gear types, in **SECTION 11**.

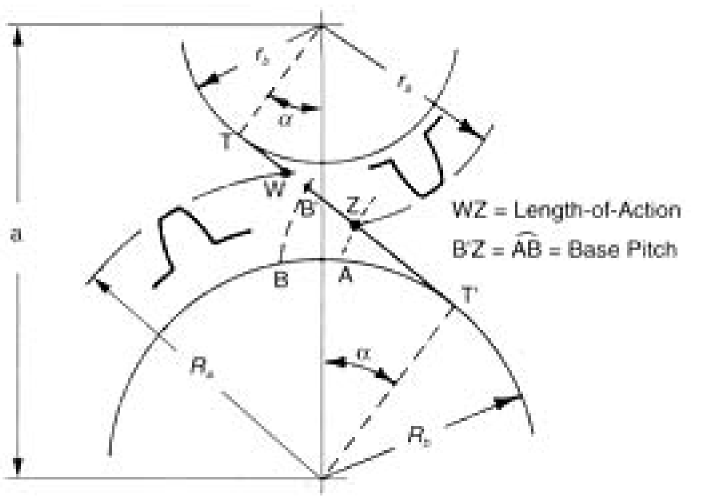

Fig. 3-3 Geometry of Contact Ratio

3.3 The Involute Function

Figure 3-4 shows an element of involute curve. The definition of involute curve is the curve traced by a point on a straight line which rolls without slipping on the circle. The circle is called the base circle of the involutes. Two opposite hand involute curves meeting at a cusp form a gear tooth curve. We can see, from **Figure 3-4**, the length of base circle arc ac equals the length of straight line bc.

$$\tan\alpha = \frac{bc}{Oc} = \frac{r_b\theta}{r_b} = \theta \text{ (radian)} \qquad (3\text{-}5)$$

The q in **Figure 3-4** can be expressed as invα + α, then **Formula (3-5)** will become:

$$\text{inv}\alpha = \tan\alpha - \alpha \qquad (3\text{-}6)$$

Function of α, or invα, is known as involute function. Involute function is very important in gear design. Involute function values can be obtained from appropriate tables. With the center of the base circle 0 at the origin of a coordinate system, the involute curve can be expressed by values of x and y as follows:

$$\left.\begin{aligned} x &= r\cos(\text{inv}\alpha) = \frac{r_b}{\cos\alpha}\cos(\text{inv}\alpha) \\ y &= r\sin(\text{inv}\alpha) = \frac{r_b}{\cos\alpha}\sin(\text{inv}\alpha) \end{aligned}\right\} \qquad \textbf{(3-7)}$$

where, $r = \frac{r_b}{\cos\alpha}$

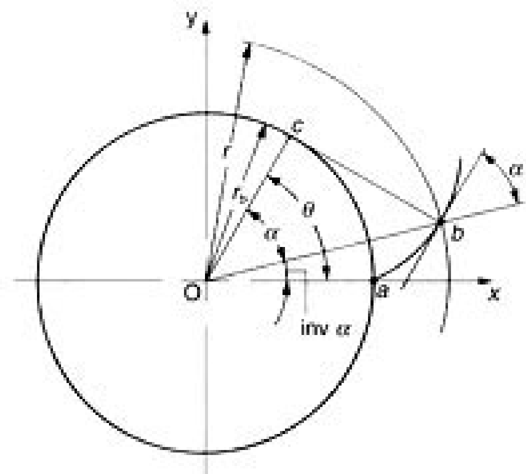

Fig. 3-4 The Involute Curve

SECTION 4 SPUR GEAR CALCULATIONS

4.1 Standard Spur Gear

Figure 4-1 shows the meshing of standard spur gears. The meshing of standard spur gears means pitch circles of two gears contact and roll with each other. The calculation formulas are in **Table 4-1**.

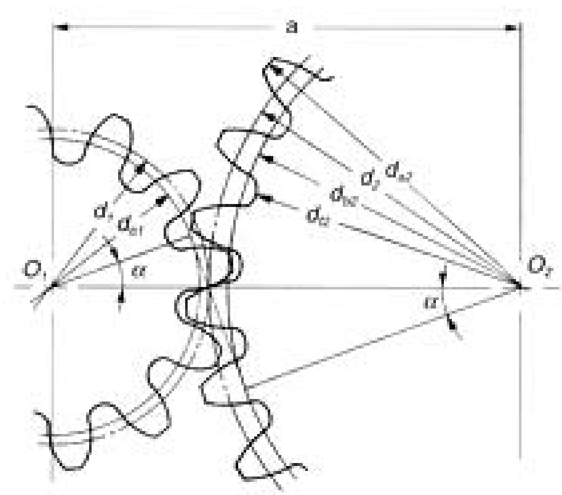

Fig. 4-1 The Meshing of Standard Spur Gears
($\alpha = 20°$, $z_1 = 12$, $z_2 = 24$, $x_1 = x_2 = 0$)

Table 4-1 The Calculation of Standard Spur Gears

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20º	
3	Number of Teeth	$z1, z2$*		12	24
4	Center Distance	a	$\frac{(z_1 + z_2)m}{2}$ *	54.000	
5	Pitch Diameter	d	zm	36.000	72.000
6	Base Diameter	d_b	$d \cos \alpha$	33.829	67.658
7	Addendum	h_a	1.00m	3.000	
8	dedendum	h_f	1.25m	3.750	
9	Outside Diameter	d_a	$d + 2m$	42.000	78.000
10	Root Diameter	d_f	$d - 2.5m$	28.500	64.500

* The subscripts 1 and 2 of z1 and z2 denote pinion and gear.

All calculated values in **Table 4-1** are based upon given module - in and number of teeth z_1 and z_2 If instead module m, center distance a and speed ratio i are given, then the number of teeth, z_1 and z_2, would be calculated with the formulas as shown in **Table 4-2**.

Table 4-2 The Calculation of Teeth Number

No.	Item	Symbol	Formula		Example	
1	Module	m			3	
2	Center Distance	a			54.000	
3	Speed Ratio	i			0.8	
4	Sum of No. of Teeth	z_1+z_2	$\frac{2a}{m}$		36	
5	Number of Teeth	z_1, z_2	$\frac{i(z_1+z_2)}{i+1}$	$\frac{(z_1+z_2)}{i+1}$	16	20

Note that the numbers of teeth probably will not be integer values by calculation with the formulas in Table 4-2. Then it is incumbent upon the designer to choose a set of integer numbers of teeth that are as close as possible to the theoretical values. This will likely result in both slightly changed gear ratio and center distance. Should the center distance be inviolable, it will then be necessary to resort to profile shifting. This will be discussed later in this section.

4.2 The Generating Of A Spur Gear

Involute gears can be readily generated by rack type cutters. The hob is in effect a rack cutter. Gear generation is also accomplished with gear type cutters using a shaper or planer machine.

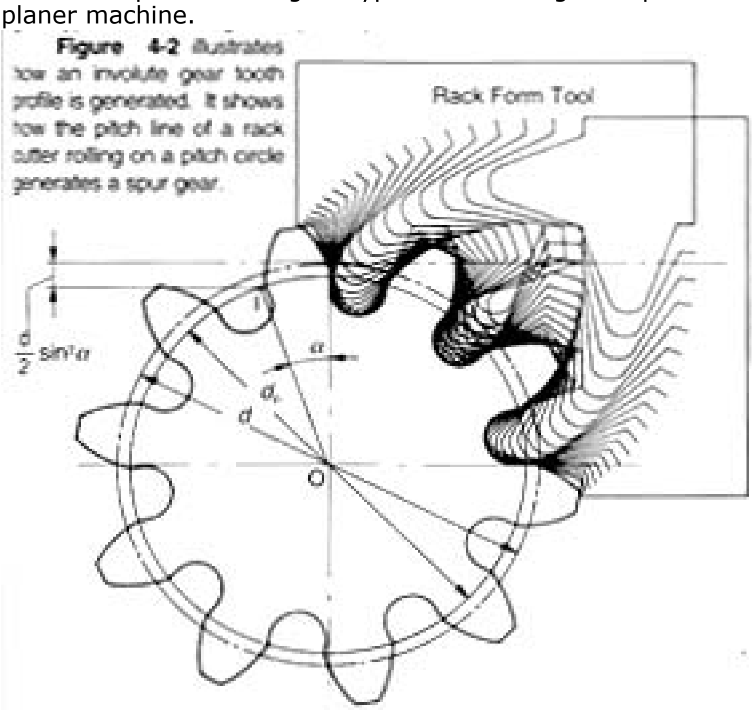

Fig. 4-2 The Generating of a Standard Spur Gear
($\alpha = 20°$, $z = 10$, $x = 0$)

4.3 undercutting

From Figure 4-3, it can be seen that the maximum length of the line-of-contact is limited to the length of the common tangent. Any tooth addendum that extends beyond the tangent points (T and T') is not only useless, but interferes with the root fillet area of the mating tooth. This results in the typical undercut tooth, shown, in **Figure 4-4**. The undercut not only weakens the tooth with a wasp-like waist, but also removes some of the useful involute adjacent to the base circle.

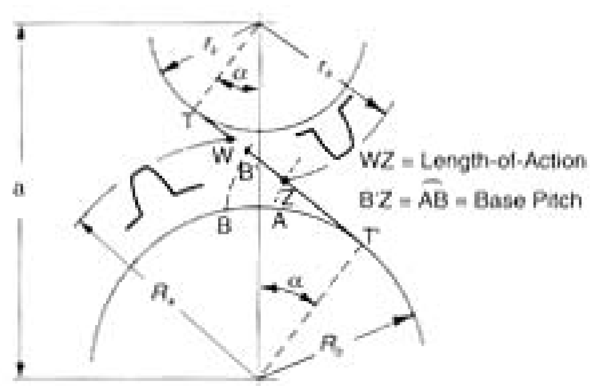

Fig. 4-3 Geometry of Contact Ratio

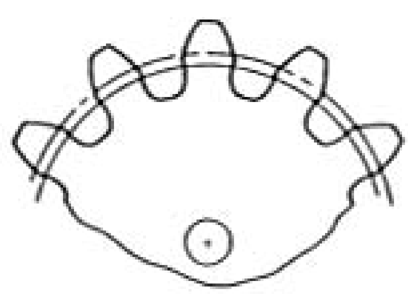

Fig. 4-4 Example of Undercut Standard Design Gear
(12 Teeth, 20° Pressure Angle)

From the geometry of the limiting length-of-contact (T-T', **Figure 4-3**), it is evident that interference is first encountered by the addenda of the gear teeth digging into the mating-pinion tooth flanks. Since addenda are standardized by a fixed value (h_a = m), the interference condition becomes more severe as the number of teeth on the mating gear increases. The limit is reached when the gear becomes a rack. This is a realistic case since the hob is a rack-type cutter. The result is that standard gears with teeth numbers below a critical value are automatically undercut in the generating process. The condition for no undercutting in a standard spur gear is given by the expression:

$$\text{Max addendum} = h_a \leq \frac{mz}{2}\sin^2\alpha$$

and the minimum number of teeth is: **(4-1)**

$$z_c \geq \frac{2}{\sin^2\alpha}$$

This indicates that the minimum number of teeth free of undercutting decreases with increasing pressure angle. For 14.5ø the value of z_C is 32, and for 20º it is 18. Thus, 20ø pressure angle gears with low numbers of teeth have the advantage of much less undercutting and, therefore, are both stronger and smoother acting.

4.4 Enlarged Pinions

Undercutting of pinion teeth is undesirable because of losses of strength, contact ratio and smoothness of action. The severity of these faults depends upon how far below z_C, the teeth number is. Undercutting for the first few numbers is small and in many applications its adverse effects can be neglected.

For very small numbers of teeth, such as ten and smaller, and for high-precision applications, undercutting should be avoided. This is achieved by pinion enlargement (or correction as often termed), wherein the pinion teeth, still generated with a standard cutter, are shifted radially outward to form a full involute tooth free of undercut. The tooth is enlarged both radially and circumferentially. Comparison of a tooth form before and after enlargement is shown in **Figure 4-5.**

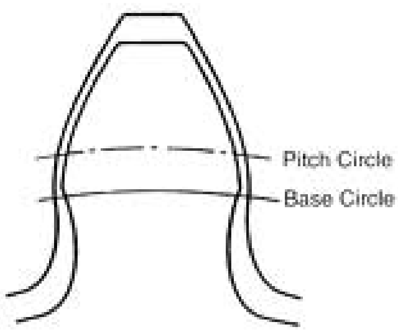

Fig. 4-5 Comparison of Enlarged and Undercut Standard Pinion
(13 Teeth, 20° Pressure Angle, Fine Pitch Standard)

4.5 Profile Shifting

As Figure 4-2 shows, a gear with 20 degrees of pressure angle and 10 teeth will have a huge undercut volume. To prevent undercut, a positive correction must be introduced. A positive correction, as in Figure 4-6, can prevent undercut

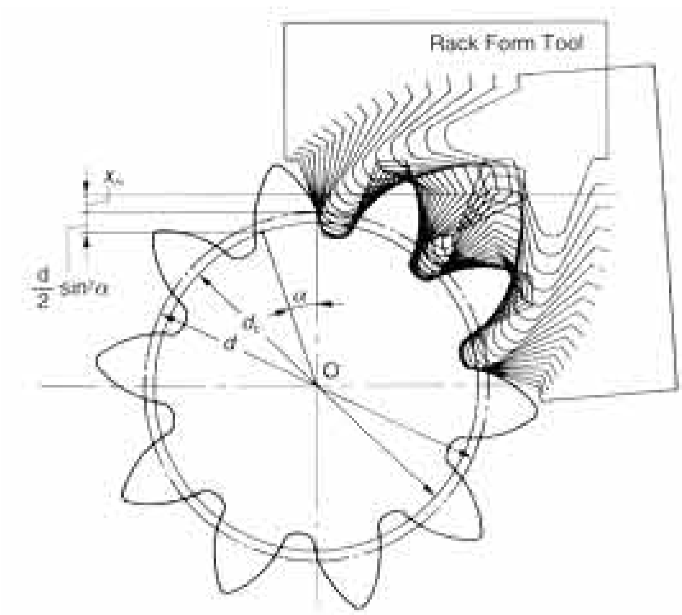

Fig. 4-6 Generating of Positive Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = +0.5$)

Undercutting will get worse if a negative correction is applied, See Figure 4-7.

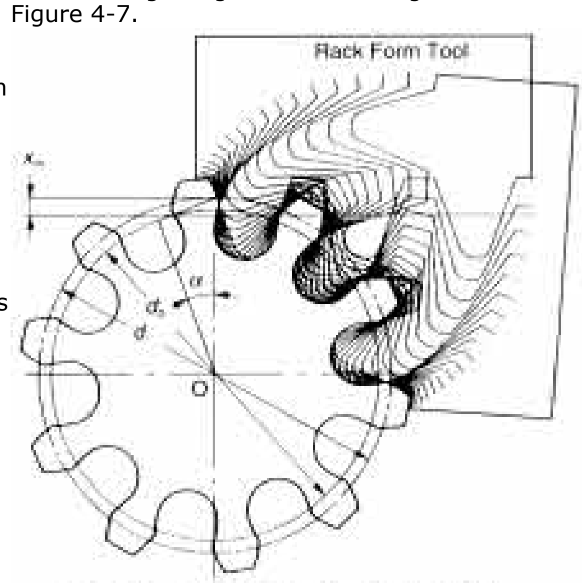

Fig. 4-7 The Generating of Negative Shifted Spur Gear
($\alpha = 20°$, $z = 10$, $x = -0.5$)

The extra feed of gear cutter (xm) in Figures 4-6 and 4-7 is the amount of shift or correction. And x is the shift coefficient.

The condition to prevent undercut in a spur gear is:

$m - xm$ £ $\frac{zm}{2} \sin^2 a$ (4-2)

The number of teeth without undercut will be:

$z_C = \frac{2(1 - x)}{\sin^2 a}$ (4 - 3)

The coefficient without undercut is:

$x = 1 - \frac{z_C 2\sin^2 a}{2}$ (4-4)

Profile shift is not merely used to prevent undercut. It can be used to adjust center distance between two gears.

If a positive correction is applied, such as to prevent undercut in a pinion, the tooth thickness at top is thinner.

Table 4-3 presents the calculation of top land thickness.

Table 4-3 The Calculations of Top Land Thickness

No.	Item	Symbol	Formula	Example
1	Pressure angle at outside circle of gear	α_a	$\cos^{-1}\left(\frac{d_b}{d_a}\right)$	m = 2, a = 20º, z = 16 x = +0.3, d = 32 d_b = 30.17016 d_a = 37.2 α_a = 36.06616º invα_a = 0.098835 invα = 0.014904 ϕ = 1.59815º (0.027893radian) S_a = 1.03762
2	Half of top land angle of outside circle	ϕ	$\frac{\pi}{2z} + \frac{2x\tan\alpha}{z} + (\text{inv}\alpha - \text{inv}\alpha_a)$ (radian)	
3	Top land thickness	S_a	θd_a	

4.6 Profile Shifted Spur Gear

Figure 4-8 shows the meshing of a pair of profile shifted gears. The key items in profile shifted gears are the operating (working) pitch diameters dw and the working (operating) pressure angle aw

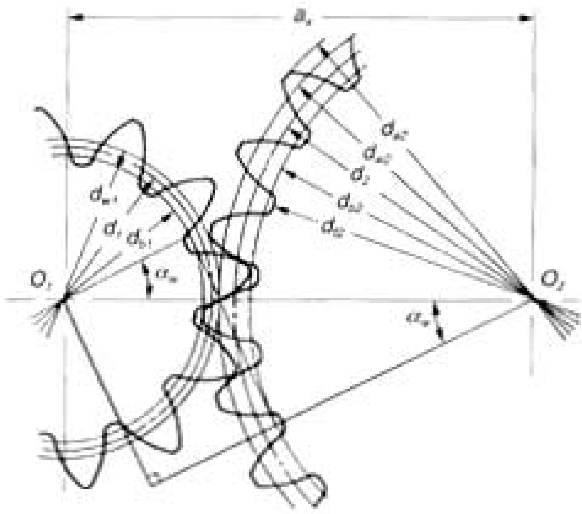

Fig. 4-8 The Meshing of Profile Shifted Gears
($\alpha = 20^\circ$, $z_1 = 12$, $z_2 = 24$, $x_1 = +0.6$, $x_2 = +0.36$)

These values are obtainable from the operating (or i.e., actua center distance and the following formulas:

$$d_{w1} = 2a_x \frac{z_1}{z_1 + z_2}$$

$$d_{w2} = 2a_x \frac{z_2}{z_1 + z_2} \qquad (4\text{-}5)$$

$$\alpha_w = \cos^{-1}\left(\frac{d_{b1} + d_{b2}}{2a_x}\right)$$

In the meshing of profile shifted gears, it is the operating pitch circles that are in contact and roll on each other that portrays gear action. The standard pitch circles no longer are of significance; and the operating pressure angle is what matters.

A standard spur gear is, according to **Table 4-4**, a profile shifted gear with 0 coefficient of shift; that is, $x_1=x_2=0$.

Table 4-5 is the inverse formula of items from 4 to 8 of **Table 4-4**.

There are several theories concerning how to distribute the sum of coefficient of profile shift, $x_1 + x_2$, into pinion, x_1, and gear, x_2, separately. BSS (British) and DIN (German) standards are the most often used. In the example above, the 12 tooth pinion was given sufficient correction to prevent undercut, and the residual profile shift was given to the mating gear.

Table 4-4 The Calculation of Positive Shifted Gear (1)

No.	Item	Symbol	Formula	Example	
				Pinion	Gear
1	Module	m		3	
2	Pressure Angle	α		20°	
3	Number of Teeth	z_1, z_2		12	24
4	Coefficient of Profile Shift	x_1, x_2		0.6	0.36
5	Involute Function α_w	inv α_w	$2\tan\alpha\left(\frac{x_1+x_2}{z_1+z_2}\right) + \text{inv}\,\alpha$	0.034316	
6	Working Pressure Angle	α_w	Find from Involute Function Table	26.0886°	
7	Center Distance Increment Factor	y	$\frac{z_1+z_2}{2}\left(\frac{\cos\alpha}{\cos\alpha_w} - 1\right)$	0.83329	
8	Center Distance	a_x	$\left(\frac{z_1+z_2}{2} + y\right)m$	56.4999	
9	Pitch Diameter	d	zm	36.000	72.000
10	Base Diameter	d_b	$d\cos\alpha$	33.8289	67.6579
11	Working Pitch Diameter	d_w	$\frac{d_b}{\cos\alpha_w}$	37.667	75.333
12	Addendum	h_{a1} h_{a2}	$(1 + y - x_2)m$ $(1 + y - x_1)m$	4.420	3.700
13	Whole Depth	h	$[2.25 + y - (x_1 + x_2)]m$	6.370	
14	Outside Diameter	d_a	$d + 2h_a$	44.840	79.400
15	Root Diameter	d_f	$d_a - 2h$	32.100	66.660

Table 4-5 The Calculation of Positive Shifted Gear (2)

No.	Item	Symbol	Formula	Example
1	Center Distance	a_x		56.4999
2	Center Distance Increment Factor	y	$\frac{a_x}{m} - \frac{z_1+z_2}{2}$	0.8333
3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{(z_1+z_2)\cos\alpha}{\ldots}\right]$	26.0886°

3	Working Pressure Angle	α_w	$\cos^{-1}\left[\frac{[illegible]}{2y + z_1 + z_2}\right]$	26.0886	
4	Sum of Coefficient of Profile Shift	$x_1 + x_2$	$\frac{(z_1 + z_2)(\text{inv}\alpha_w - \text{inv}\,\alpha)}{2\tan\alpha}$	0.9600	
5	Coefficient of Profile Shift	$x_1\ ,\ x_2$		0.6000	0.3600